DIRECTORATE OF EDUCATION
GNCT of Delhi, Delhi Government

SUPPORT MATERIAL
(2020-2021)
Class : X

MATHEMATICS

Under the Guidance of

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IAS

सत्यमेव जयते

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MESSAGE

The importance of adequate practice during examinations can never be overemphasized. I am happy that support material for classes IX to XII has been developed by the Examination Branch of Directorate of Education. This material is the result of immense hard work, co-ordination and cooperation of teachers and group leaders of various schools. The purpose of the support material is to impart ample practice to the students for preparation of examinations. It will enable the students to think analytically & rationally, and test their own capabilities and level of preparation.

The material is based on latest syllabus prepared by the NCERT and adopted by the CBSE for the academic session 2020-21 and covers different levels of difficulty. I expect that Heads of Schools and Teachers will enable and motivate students to utilize this material during zero periods, extra classes and regular classes best to their advantage.

I would like to compliment the team of Examination Branch for their diligent efforts of which made it possible to accomplish this work in time. I also take this opportunity to convey my best wishes to all the students for success in their endeavours.

(Manisha Saxena)

BINAY BHUSHAN, IAS

सत्यमेव जयते

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Dear Students,

Directorate of Education is committed to providing qualitative and best education to all its students. The Directorate is continuously engaged in the endeavor to make available the best study material for uplifting the standard of its students and schools.

Every year, the expert faculty of Directorate reviews and updates Support Material. The expert faculty of different subjects incorporates the changes in the material as per the latest amendments made by CBSE to make its students familiar with new approaches and methods so that students do well in the examination.

The book in your hand is the outcome of continuous and consistent efforts of senior teachers of the Directorate. They have prepared and developed this material especially for you. A huge amount of money and time has been spent on it in order to make you updated for annual examination.

Last, but not the least, this is the perfect time for you to build the foundation of your future. I have full faith in you and the capabilities of your teachers. Please make the fullest and best use of this Support Material.

BINAY BHUSHAN
DIRECTOR (EDUCATION)

Dr. (Mrs.) Saroj Bala Sain
Addl. Director of Education
(School / Exam / EVGB/IEB/ VOC.)

सत्यमेव जयते

Govt. of NCT of Delhi
Directorate of Education
Old Secretariat, Delhi-110054
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D.O. No. PA/Addl.DE(Sch)/86
Date : 03-10-2019

I am very much pleased to forward the Support Material for classes IX to XII. Every year, the Support Material of most of the subjects is updated/revised as per the most recent changes made by CBSE. The team of subject experts, officers of Exam Branch, members of Core Academic Unit and teachers from various schools of Directorate has made it possible to make available unsurpassed material to students.

Consistence use of Support Material by the students and teachers will make the year long journey seamless and enjoyable. The main purpose to provide the Support Material for the students of government schools of Directorate is not only to help them to avoid purchasing of expensive material available in the market but also to keep them updated and well prepared for exam. The Support Material has always been a ready to use material, which is matchless and most appropriate.

I would like to congratulate all the Team Members for their tireless, unremitting and valuable contributions and wish all the best to teachers and students.

S. Sain

(Dr. Saroj Bala Sain)
Addl.DE (School/Exam)

DIRECTORATE OF EDUCATION
Govt. of NCT, Delhi

SUPPORT MATERIAL
(2020-2021)

MATHEMATICS

Class : X
(English Medium)

NOT FOR SALE

PUBLISHED BY : DELHI BUREAU OF TEXTBOOKS

Team Members for Review of Support Material

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भारत का संविधान

भाग 4क

नागरिकों के मूल कर्तव्य

अनुच्छेद 51क

मूल कर्तव्य – भारत के प्रत्येक नागरिक का यह कर्तव्य होगा कि वह –

1. संविधान का पालन करे और उसके आदर्शों, संस्थाओं, राष्ट्र ध्वज और राष्ट्रगान का आदर करें।
2. स्वतंत्रता के लिए हमारे राष्ट्रीय आंदोलन को प्रेरित करने वाले उच्च आदर्शों को हृदय में संजोए रखे और उनका पालन करे।
3. भारत की प्रभुता, एकता और अखंडता की रक्षा करे और उसे अक्षुण्ण रखे।
4. देश की रक्षा करे।
5. भारत के सभी लोगों में समरसता और समान भ्रातृत्व की भावना का निर्माण करे।
6. हमारी सामाजिक संस्कृति की गौरवशाली परंपरा का महत्त्व समझे और उसका निर्माण करे।
7. प्राकृतिक पर्यावरण की रक्षा और उसका संवर्धन करे।
8. वैज्ञानिक दृष्टिकोण और ज्ञानार्जन की भावना का विकास करे।
9. सार्वजनिक संपत्ति को सुरक्षित रखे।
10. व्यक्तिगत एवं सामूहिक गतिविधियों के सभी क्षेत्रों में उत्कर्ष की ओर बढ़ने का सतत् प्रयास करे।
11. माता–पिता या संरक्षक द्वारा 6 से 14 वर्ष के बच्चों हेतु प्राथमिक शिक्षा प्रदान करना (86वां संशोधन)।

CONSTITUTION OF INDIA

Part IV A (Article 51 A)

Fundamental Duties

Fundamental Duties : It shall be the duty of every citizen of India —

1. to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;
2. to cherish and follow the noble ideals which inspired our national struggle for freedom;
3. to uphold and protect the sovereignty, unity and integrity of India;
4. to defend the country and render national service when called upon to do so;
5. to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;
6. to value and preserve the rich heritage of our composite culture;
7. to protect and improve the natural environment including forests, lakes, rivers and wild life, and to have compassion for living creatures.
8. to develop the scientific temper, humanism and the spirit of inquiry and reform;
9. to safeguard public property and to adjure violence;
10. to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement.
11. who is a parent or guardian to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

भारत का संविधान

उद्देशिका

हम, भारत के लोग, भारत को एक **(सम्पूर्ण प्रभुत्व–सम्पन्न समाजवादी पंथनिरपेक्ष लोकतंत्रात्मक गणराज्य)** बनाने के लिए, तथा उसके समस्त नागरिकों को :

सामाजिक, आर्थिक और राजनैतिक **न्याय,**

विचार, अभिव्यक्ति, विश्वास, धर्म

और उपासना की **स्वतंत्रता,**

प्रतिष्ठा और अवसर की समता

प्राप्त करने के लिए,

तथा उन **सब में,**

व्यक्ति की गरिमा और **(राष्ट्र की एकता**

और अखंडता) सुनिश्चित करने वाली बंधुता

बढ़ाने के लिए

हम दृढ़संकल्प होकर इस संविधान को आत्मार्पित करते हैं।

THE CONSTITUTION OF INDIA

PREAMBLE

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a **(SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC)** and to secure to all its citizens :

JUSTICE, social, economic and political,

LIBERTY of thought, expression, belief, faith and worship,

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the **(unity an integrity of the Nation)**;

WE DO HEREBY GIVE TO OURSELVES THIS CONSTITUTION.

COURSE STRUCTURE CLASS -X

Units	Unit Name	Marks
I	NUMBER SYSTEMS	06
II	ALGEBRA	20
III	COORDINATE GEOMETRY	06
IV	GEOMETRY	15
V	TRIGONOMETRY	12
VI	MENSURATION	10
VII	STATISTICS & PROBABILTY	11
	Total	**80**

UNIT I: NUMBER SYSTEMS

1. REAL NUMBER

Euclid's division lemma, Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of $\sqrt{2}, \sqrt{3}, \sqrt{5}$ Decimal representation of rational numbers interms of terminating/non-terminating recurring decimals.

UNIT II: ALGEBRA

1. POLYNOMIALS

Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials. Statement and simple problems on division algorithm for polynomials with real coefficients.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency.

Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination and by cross multiplication method. Simple situational problems. Simple problems on equations reducible to linear equations.

3. QUADRATIC EQUATIONS

Standard form of a quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$. Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.

Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. ARITHMETIC PROGRESSIONS

Motivation for studying Arithmetic Progression Derivation of the n^{th} term and sum of the first n terms of A.P. and their application in solving daily life problems.

UNIT III: COORDINATE GEOMETRY

1. LINES (In two-dimensions)

Review: Concepts of coordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division). Area of a triangle.

UNIT IV: GEOMETRY

1. TRIANGLES

Definitions, examples, counter examples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.
5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.
6. (Motivate) If a perpendicular is drawn from the vertex of the right angle of a right triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.
7. (Prove) The ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.
8. (Prove) In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
9. (Prove) In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angles opposite to the first side is a right angle.

2. CIRCLES

Tangent to a circle at, point of contact

1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.

3. CONSTRUCTIONS

1. Division of a line segment in a given ratio (internally).
2. Tangents to a circle from a point outside it.
3. Construction of a triangle similar to a given triangle.

UNIT V: TRIGONOMETRY

1. INTRODUCTION TO TRIGONOMETRY

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0° and 90°. Values of the trigonometric ratios of 30^0, 45^0 and 60^0. Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES

Proof and applications of the identity $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given.Trigonometric ratios of complementary angles.

3. HEIGHTS AND DISTANCES: Angle of elevation, Angle of Depression.

Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30°, 45°, 60°.

UNIT VI: MENSURATION

1. AREAS RELATED TO CIRCLES

Motivate the area of a circle; area of sectors and segments of a circle. Problems based on areas and perimeter / circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only. Plane figures involving triangles, simple quadrilaterals and circle should be taken.)

2. SURFACE AREAS AND VOLUMES

1. Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones. Frustum of a cone.
2. Problems involving converting one type of metallic solid into another and other mixed problems. (Problems with combination of not more than two different solids be taken).

UNIT VII: STATISTICS AND PROBABILITY

1. STATISTICS

Mean, median and mode of grouped data (bimodal situation to be avoided). Cumulative frequency graph.

2. PROBABILITY

Classical definition of probability. Simple problems on finding the probability of an event.

MATHEMATICS-Standard
Code (041)
QUESTION PAPER DESIGN
CLASS – X (2020-21)

Time : 3 Hours **Max. Marks: 80**

S. No.	Typology of Questions	Very Short Answer-Objective type (VSA) (1 Mark)	Short Answer-I (SA) (2 Marks)	Short Answer-II (SA) (3 Marks)	Long Answer (LA) (4 Marks)	Total Marks	% Weightage (approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers.	6	2	2	1	20	25
2	**Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	6	1	1	3	23	29
3	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	5	2	2	1	19	24
4	**Analyzing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	3	1	3	1	18	22
	Total	20x1 =20	6x2 =12	8x3=24	6x4=24	80	100

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

MATHEMATICS-Basic
Code (241)
QUESTION PAPER DESIGN
CLASS – X (2020-21)

Time : 3Hours **Max. Marks: 80**

S. No.	Typology of Questions	Very Short Answer-Objective type (VSA) (1 Mark)	Short Answer-I (SA) (2 Marks)	Short Answer-II (SA) (3 Marks)	Long Answer (LA) (4 Marks)	Total Marks	% Weightage (approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers.	5	2	5	2	32	40
2	**Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	7	1	1	4	28	35
3	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	5	2	1	-	12	15
4	**Analyzing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	3	1	1	-	8	10
	Total	20x1 =20	6x2 =12	8x3=24	6x4=24	80	100

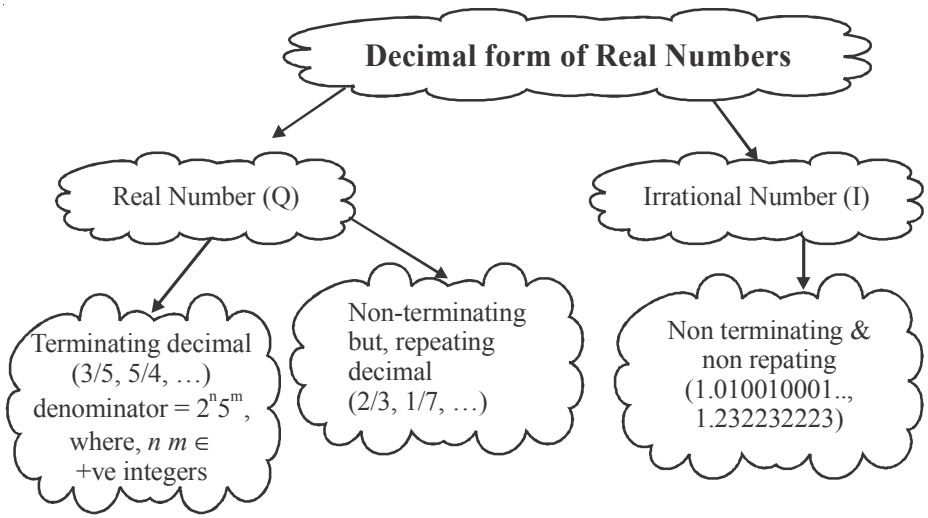
INTERNAL ASSESSMENT	20 MARKS
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

Content

CHAPTER

1 Real Numbers

KEY POINTS



PROPERTIES OF REAL NUMBERS

Euclid division Lemma
given +ve integers a & b, there exist unique integers q & r satisfying $a = bq + r$, $0 \le r < b$.

Fundamental Theorem of Airthmatic
Every composite number can be expressed factorised as a product of primes and this factorisation is unique apart from the order in which the prime factors occur.

Euclid division algorithm

The HCF of any two +ve integers a & b with $a > b$, is obtained as follows:

Step 1 : Apply Euclid division lemma to a & b to find q & r, where a = bq + r , $0 \le r < b$.

Step 2 : If r = 0, the HCF is b If r ¹ 0 then apply Euclid 's lemma to b & r to find b_1, and r_1, where $b = b_1\ r + r_1$

Step 3 : If $r_1 = 0$, HCF is b, If r_1 ¹ 0, then continue the process. till $r_n = 0$ then bn at this stage will be HCF.

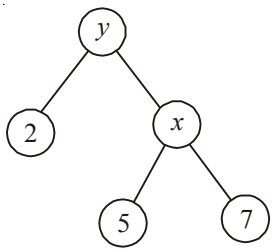
VERY SHORT ANSWER TYPE QUESTIONS

1. A number N when divided by 16 gives the remainder 5 _____ is the remainder when the same number is divided by 8.

2. HCF of $3^3 \times 5^4$ and $3^4 \times 5^2$ is _____ .

3. If $a = xy^2$ and $b = x^3y^5$ where x and y are prime numbers then LCM of (a, b) is _____ .

4. In factor tree find x and y



5. If n is a natural number, then $25^{2n} - 9^{2n}$ is always divisible by :

(i) 16 (ii) 34

(iii) both 16 or 34 (iv) None of these

6. The decimal expansion of the rational number $\frac{327}{2^3 \times 5}$ will terminate after

(a) One decimal place (b) Two decimal place

(c) Three decimal place (d) More than three decimal place

7. Which of the following rational numbers have terminating decimal?

(i) $\frac{16}{225}$ (ii) $\frac{5}{18}$ (iii) $\frac{2}{21}$ (iv) $\frac{7}{250}$

(a) (i) and (iii) (b) (ii) and (iii)

(c) (i) and (iii) (d) (i) and (iv)

8. Euclid's division Lemma states that for two positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must satisfy.

(a) $1 < r < b$ (b) $0 < r \leq b$

(c) $0 \leq r < b$ (d) $0 < r < b$

9. $p^n = (a \times 5)^n$ For p^n to end with the digit zero $a =$ _____ for natural number n.

(a) any natural number (b) even number

(b) odd number (d) none of these

10. HCF is always

(a) multiple of LCM (b) Factor of LCM

(c) divisible by LCM (d) a and c both

11. All decimal numbers are

(a) rational number (b) irrational numbers

(c) real numbers (d) integers

12. Which of these numbers always end with the digits 6.

(a) 4^n (b) 2^n (c) 6^n (d) 8^n

13. Write the general form of an even integer

14. Write the form in which every odd integer can be written taking t as variable.

15. What would be the value of n for which n^2-1 is divisible by 8.

16. What can you say about the product of a non-zero rational and irrational number?

17. After how many places the decimal expansion of $\frac{13497}{1250}$ will terminate?

18. Find the least number which is divisible by all numbers from 1 to 10 (both inclusive).

19. The numbers 525 and 3000 are divisible by 3, 5, 15, 25 and 75 what is the HCF of 525 and 3000?

20. What will be the digit at unit's place of 9^n?

SHORT ANSWER TYPE QUESTIONS-I

21. If n is an odd integer then show that $n^2 - 1$ is divisible by 8.

22. Use Euclid's division algorithm to find the HCF of 16 and 28.

23. Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

(NCERT Exemplar)

24. Without actual performing the long division, find if $\frac{395}{10500}$ will have terminating or non terminating (repeating decimal expansion.)

25. A rational number in its decimal expansion is 327. 7081. What can you say about the prime factors of q, when this number is expressed in the form of $\frac{p}{q}$? Give reasons.

26. What is the smallest number by which $\sqrt{5}-\sqrt{2}$ is to be multiplied to make it a rational number? Also find the number so obtained?

27. Find one rational and one irrational no between $\sqrt{3}$ and $\sqrt{5}$.

28. If HCF of 144 and 180 is expressed in the form 13m – 3, find the value of m. (CBSE 2014)

29. Find the value of : $(-1)^n + (-1)^{2n} + (-1)^{2n+1} + (-1)^{4n+2}$, where n is any positive and integer. (CBSE : 2016)

30. Show that any positive add integer is of the form 4q + 1 or 4q + 3, where q is some integer. (CBSE : 2012)

31. Two tankers contain 850 litres and 680 litres of petrol respectively. Find the maximum capacity of a container which can measure the petrol of either tanker in exact number of times. (CBSE : 2016)

SHORT ANSWER TYPE QUESTIONS-II

32. Show that the cube of any positive integer is of the form $4m$, $4m + 1$ or $4m + 3$ for some integer m.

33. Prove that $\sqrt{3}$ is an irrational number.

34. State fundamental theorem of Arithmetic and hence find the unique factorization of 120.

35. Prove that $\sqrt{3} + \sqrt{5}$ is irrational

36. Prove that $5 - \frac{3}{7}\sqrt{3}$ is an irrational number.

37. Prove that $\frac{1}{2-\sqrt{5}}$ is an irrational number.

38. Find HCF and LCM of 56 and 112 by prime factorization method.

39. Explain why:

(i) $7 \times 11 \times 13 \times 15 + 15$ is a composite number

(ii) $11 \times 13 \times 17 + 17$ is a composite number.

(iii) $1 \times 2 \times 3 \times 5 \times 7 + 3 \times 7$ is a composite number.

40. On a morning walk, three perosns steps off together and their steps measure 40 cm, 42 cm, and 45 cm respectively. What is the minimum distance each should walk, so that each can cover the same distance in complete steps? (NCERT Exemplar)

41. During a sale, colour pencils were being sold in the pack of 24 each and crayons in the pack of 32 each. If you want full packs of both and the same number of pencils and crayons, how many packets of each would you need to buy? (CBSE : 2017)

42. Find the largest number that divides 31 and 99 leaving remainder 5 and 8 respectively.

43. The HCF of 65 and 117 is expressible in the form 65 m – 117. Find the value of m. Also find the LCM of 65 and 117 using prime factorisation method.

44. Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainder 1, 2 and 3 respectively. (NCERT Exemplar)

45. Show that square of any odd integer is of the form 4m + 1, for some integer m.

46. Find the HCF of 180, 252 and 324 by Euclid's Division algorithm.

47. Find the greatest number of six digits exactly divisible by 18, 24 and 36.

48. Three bells ring at intervals of 9, 12, 15 minutes respectively. If they start ringing together at a time, after what time will they next ring together?

49. Show t hat only one of the number of n, $n + 2$ and $n + 4$ is divisble by 3.

50. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of two given number. (CBSE : 2018)

LONG ANSWER TYPE QUESTIONS

51. Find the HCF of 56, 96, 324 by Euclid's algorithm.

52. Show that any positive odd integer is of the form $6q + 1$, $6q + 3$ or $6q + 5$, where q is some integer.

53. Prove that the square of any positive integer is of the form $5q$, $5q + 1$, $5q + 4$ for some integer, q.

54. Prove that the product of three consecutive positive integers is divisible by 6.

55. For any positive integer n, prove that $n^3 – n$ is divisible by 6. (NCERT Exemplar)

56. Show that one and only one of n, $n + 2$, $n + 4$ is divisible by 3.

57. Aakriti decided to distribute milk in an orphanage on her birthday. The supplier brought two milk containers which contain 398 *l* and 436 *l* of milk. The milk is to be transferred to another containers so that 7 *l* and 11 *l* of milk is left in both the containers respectively. What will be the maximum capacity of the drum?

58. Find the smallest number, which when increased by 17, is exactly divisible by both 520 and 468.

59. A street shopkeeper prepares 396 Gulab jamuns and 342 ras-gullas. He packs them, in combination. Each containter consists of either gulab jamuns or ras-gullab but have equal number of pieces.

Find the number of pieces he should put in each box so that number of boxes are least. (CBSE 2016)

60. Show that the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for integer q.

61. Express the HCF of numbers 72 and 124 as a linear combination of 72 and 124.

62. Show that there is no positive integer n for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.

63. Find the HCF of numbers 134791, 6341 and 6339 by Euclid's div ision algorithm.

64. In a seminar, the no. of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively. Find the minimum number of rooms required if in each room the some the same number of participants are to be seated and all of the them being of the the same subject. (HOTS)

65. State fundamental theorem of Arithmetic. Is it possible that HCF and LCM of two numbers be 24 and 540 respectively. Justify your answer.

ANSWERS AND HINTS

1. 5

2. $3^3 \times 5^2$

3. $x^3 \times y^5$

4. $x = 35, y = 70$

5. (*iii*) $25^{2n} - 9^{2n}$ is of the form $a^{2n} - b^{2n}$ which is divisible by both $a - b$ and $a + b$ so, by both $25 + 9 = 34$ and $25 - 9 = 16$.

6. (c) three decimal place

7. (d) (i) and (iv)

8. (c) $0 \le r < b$

9. (b) even number

10. (b) Factor of LCM

11. (c) real numbers

12. (c) 6^n

13. 2m

14. $2t + 1$

15. An odd integer

16. Irrational

17. 4

18. 2520

19. 75

20. 1 and 9

21. Any +ve odd integer is of the form 4q + 1 or 4q + 3 for some integer q so if $n = 4q + 1$.

$n^2 - 1 = (4q + 1)^2 - 1 = 16q^2 + 8q = 8q(2q + 1) \Rightarrow n^2 - 1$ is divisible by 8.

If $n = 4q + 3$

$n^2 - 1 = (4q + 3)^2 - 1 = 16q^2 + 24q + 8 = 8\,(2q^2 + 3q + 1) \Rightarrow n^2 - 1$ is divisible by 8.

22. 4

23. As 12 has factors 2, 2, 3 it doesnot has 5 as its factor so 12^n will never end with 0 or 5.

24. Non-terminating repeating.

25. Denominator is the multiple of 2's and 5's.

26. $\sqrt{5} + \sqrt{2}$, 3

28. By Euclid's division lemma

$$180 = 144 \times 1 + 36$$
$$144 = 36 \times 4 + 0$$

HCF of 180 and 144 is 36.

29. Given that n is a positive odd integer

$\Rightarrow$ $2n$ and $4n + 2$ are even positive integers and n and $2n + 1$ are odd positive integers.

$\therefore$ $(-1)^n = -1, (-1)^{2n} = +1, (-1)^{2n+1} = -1, (-1)^{2n+2} = +1$

$\therefore$ $(-1)^n + (-1)^{2n} + (-1)^{2n+1} + (-1)^{4n+2} = -1 + 1 - 1 + 1 = 0$

30. By applying Euclid division algorithm to a and b such that a = 4q + r, where $b = 4$, Now $r = 0, 1, 2, 3$.

where, $r = 0$, $a = 4q$ which is even number.

where, $r = 1$, $a = 4q + 1$ an odd number.

where, $r = 2$, $a = 4q + 2 = 2\,(2q + 1)$, an even number.

where, $r = 3$, $a = 4q + 3$ an odd number.

31. HCF of 850 and 680 is $2 \times 5 \times 17 = 170$ litres.

32. Let n be any psoitve integer. Then it is of the form $4q$, $4q + 1$, $4q + 2$ and $4q + 3$.

When $n = 4q$, $n^3 = 64q^3 = 4\,(16q^3) = 4\,m$, where $m = 16q^3$

When $n = 4q + 1$, $n^3 = (4q + 1)^3 = 64q^3 + 48q^2 + 12q + 1$

$$= 4(16q^3 + 12q^2 + 3q) + 1 = 4\,m + 1.$$

where $m = 16q^3 + 12q^2 + 3q$

Similarly discuss for $n = 4q + 2$ and $4q + 3$.

34. $2 \times 2 \times 2 \times 3 \times 5$

35. Prove that $\sqrt{3}$ and $\sqrt{5}$ is irrational number separately and sum of two irrational number is an irrational number.

36. 5 is rational no. and $\frac{3}{7}\sqrt{3}$ is an irrational number. Difference of a rational number and irrational number is an irrational number.

38. HCF : 56, LCM : 112

39. (1) $15 \times (7 \times 11 \times 13 + 1)$ as it has more than two factors so it is composite no.

40. LCM of 40, 42, 45 = 2520

Minimum distance each should walk 2520 cm.

41. LCM of 24 and 32 is 96

96 crayons or $\frac{96}{32}$ = 3 packs of crayons

96 pencils or $\frac{96}{24}$ = 4 packs of pencils.

42. Given number = 31 and 99

$31 - 5 = 26$ and $99 - 8 = 91$

Prime factors of $26 = 2 \times 13$

$91 = 7 \times 13$

HCF of (26, 91) = 13.

$\therefore$ 13 is the largest number which divides 31 and 99 leaving remainder 5 and 8 respectively.

43. HCF of 117 and 65 by Euclid division algorithm.

$117 = 65 \times 1 + 52$

$65 = 52 \times 1 = 52$

$52 = 13 \times 4 + 0$

HCF (117, 52) = 13.

Given that $65\ m - 117 = 13 \Rightarrow 65\ m = 130 \Rightarrow m = 2$.

LCM $(65, 117) = 13 \times 3^2 \times 5 = 585$

44. 1251 – 1 =1250, 9377 – 2 = 9375, 15628 – 3 = 15625

HCF of (15625, 9375) = 3125

HCF of (3125, 1250) = 625

$\Rightarrow$ HCF of (1250, 9375, 15625) = 625

45. By Euclid's division algorithm, we have $a = bq + r$, where $0 \le r < 4$. On putting $b = 4$ we get $a = 4q + r$ where, $r = 0, 1, 2, 3$.

If $r = 0$, $a = 4q$ which is even

If $r = 1$, $a = 4q + 1$ not divisible by 2

If $r = 2$, $a = 4q + 2 = 2(2q + 1)$ which is even

If $r = 3$, $a = 4q + 3$ not divisible by 2.

So, for any +ve integer q, $4q + 1$ and $4q + 3$ are odd integers.

How, $a^2 = (4q + 1)^2 = 16q^2 + 1 + 8q = 4(4q^2 + 2q) + 1 = 4m + 1$ where $m = 4q^2 + 2q$ similarly for $4q + 3$.

46. HCF (324, 252, 180) = 36

47. LCM of (18, 24, 36) = 72.

Greatest six digit number = 999999

```
72 ) 999999 ( 13888
   - 72
     279
   - 216
     639
   - 576
     639
   - 576
     639
     576
      63
```

Require six digit number

```
  999999
    - 63
  999936
```

48. LCM of (9, 12, 15) = 180 minutes.

49. Let the number divisible by 3 is of the form $3k + r$, $r = 0, 1, 2$

$a = 3k, 3k + 1$ or $3k + 2$

(*i*) When $a = 3k$

$n = 3k \Rightarrow n$ is divisible by 3.

$n + 2 = 3k + 2 \Rightarrow n + 2$ is not divisible by 3.

$n + 4 = 3k + 4 = 3k + 3 + 1 = 3(k + 1) + 1 \Rightarrow n + 4$ is not divisible by 3.

So, only one out of n, $n+2$ and $n+4$ is divisible by 3.

(*ii*) When $a = 3k + 1$

$n = 3k + 1 \Rightarrow n$ is not divisible by 3.

$n + 2 = 3k + 1 + 2 = 3k + 3 = 3(k + 1)$

$\Rightarrow n + 2$ is divisible by 3.

$n + 4 = 3k + 1 + 4 = 3k + 5 = 3(k + 1) + 2$

$\Rightarrow n + 2$ is not divisible by 3.

So, only one out of n, $n+2$ and $n+4$ is divisible by 3.

Similarly do for $a = 3k + 2$.

50. HCF (404, 96) = 4

LCM (404, 96) = 9696

HCF × LCM = 38, 784

Also, $404 \times 96 = 38{,}784$

51. 4

52. Let a be +ve odd integer, divide it by 6 then q is the quotient and r is the remiander.

$\Rightarrow$ $a = 6q + r$ where $r = 0, 1, 2, 3, 4, 5$

If, $a = 6q + 0 = 2(3q)$ is an even integer so not possible

If, $a = 6q + 1$ is an odd integer

If, $a = 6q + 2 = 2(3q + 1)$ is an even integer so not possible

If, $a = 6q + 3$ is an odd integer

If, $a = 6q + 4 = 2(3q + 2)$ is an even integer so not possible

If, $a = 6q + 5$ is an odd integer.

54. Let the three consecutive integers be a, a+1, a + 2,

Case I : If a is even,

$\Rightarrow$ $a + 2$ is the also even

$a(a + 2)$ is divisible by 2

a(a + 2) $(a + 1)$ is also divisible by 2

Now a, a + 1, a + 2 are three consecutive numbers

$\Rightarrow$ $a\,(a + 1)\,(a + 2)$ is a multiple by 3

$\Rightarrow$ $a\,(a + 1)\,(a + 2)$ is divisible by 3

as it is divisible by 2 and 3 hence divisible by 6.

Case II : If a is odd

$\Rightarrow$ $a + 1$ is even

$\Rightarrow$ $a + 1$ is divisible by 2

$\Rightarrow$ $a(a + 1)(a + 2)$ is also divisible by 2

Again a, $a + 1$, a + 2 are three consecutive numbers

$\Rightarrow$ $a(a + 1)(a + 2)$ is a multiple by 3

$\Rightarrow$ $a(a + 1)(a + 2)$ is divisible by 3

as it is divisible by 2 and 3 hence divisible by 6.

55. $n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1)$

$= (n - 1)(n)(n + 1)$

= Product of three consecutive +ve integers

Now to show that produce of three consecutive +ve integers is divisible by 6.

Any +ve integer a is of the form $3q$, $3q + 1$ or $3q + 2$ for some integer q.

Let a, $a + 1$, $a + 2$ be any three consecutive integers.

Case I : $a = 3q$

$(3q)(3q + 1)(3q + 2) = 3q$ (2m) [as (3q + 1) and (3q + 2) are consecutive integers so their product is also even]

$= 6q$ m

which is divisible by 6.

Case II : If $a = 3q + 1$

$a(a + 1)(a + 2) = (3q + 1)(3q + 2)(3q + 3)$

$= 2m^3(q + 1)$ (as $(3q + 1)(3q + 2) = 2m$)

$= 6$ m $(q + 1)$

which is divisible by 6.

Case III : If $a = 3q + 2$

$a(a + 1)(a + 2) = (3q + 2)(3q + 3)(3q + 4)$

$= (3q + 2)\, 3(q + 1)(3q + 4)$

$= 6$ m

which is divisible by 6.

57. 17

58. 4663

59. HCF (396, 342) = 18

61. HCF (124, 72) = 4

$$4 = 124 \times 7 + 72 \times (-12),\ x = 7,\ y = -12$$

62. Let $\sqrt{n-1} + \sqrt{n+1} = \frac{p}{q}$ (1)$q \neq 0$, p, q, co-prime.

$$\frac{q}{p} = \frac{1}{\sqrt{n-1} + \sqrt{n+1}} \times \frac{\sqrt{n-1} - \sqrt{n+1}}{\sqrt{n-1} - \sqrt{n+1}}$$

$$\frac{q}{p} = \frac{\sqrt{n-1} - \sqrt{n+1}}{-2}$$

$$\sqrt{n-1} + \sqrt{n+1} = -\frac{2q}{p} \text{ or } \sqrt{n+1} - \sqrt{n-1} = \frac{2q}{p} \quad \ldots(2)$$

Adding (1) & (2) we get $2\sqrt{n+1} = \frac{p}{q} + \frac{2q}{p} = \frac{p^2 + 2q^2}{pq}$ …(3)

Subtracting (1) & (2) we get $2\sqrt{n-1} = \frac{p^2 - 2q^2}{pq}$ …(4)

From (3) & (4) we get $\sqrt{n+1} + \sqrt{n-1}$ are rational numbers.

But $\sqrt{n-1} + \sqrt{n+1}$ is an irrational number.

$\therefore$ These exist no positive integer n, for which $\sqrt{n-1} + \sqrt{n+1}$ is rational.

63. HCF (134791, 6341, 6339) = 1.

64. HCF of 60, 84 and 108 is $2^2 \times 3 = 12$

$$\text{No. of rooms required} = \frac{\text{Total number of participants}}{12}$$

$$= \frac{60 + 84 + 108}{12} = 21 \text{ rooms}$$

65. HCF = 24, LCM = 540

$$\frac{\text{LCM}}{\text{HCF}} = \frac{540}{24} = 22.5, \text{ not an integer.}$$

Hence two numbers cannot have HCF and LCM as 24 and 540 respectively.

PRACTICE-TEST

Real Number

Time : 1 Hr. *M.M. : 20*

SECTION A

1. After how many decimal places the decimal expansion of $\frac{51}{150}$ will terminate. 1

2. In Euclid's Division Lemma, when $a = bq + r$ where a, b are positive integers then what values r can take? 1

3. HCF of x^4y^5 and x^8y^3 . 1

4. LCM of 14 and 122 . 1

SECTION B

5. Show that 9^n can never ends with unit digit zero. 2

6. Without actual division find the type of decimal expansion of $\frac{935}{10500}$ 2

7. Show that the square of any odd integer is of the form $4m + 1$, for some integer m. 2

SECTION C

8. Prove that $\frac{1}{3-2\sqrt{5}}$ is an irrational number. 3

9. Find the HCF of 36, 96 and 120 by Euclid's Lemma. 3

SECTION D

10. Once a sports goods retailer organized a campaign "Run to remember" to spread awareness about benefits of walking. In that Soham and Baani participated. There was a circular path around a sports field. Soham took 12 minutes to drive one round of the field, while Baani took 18 minutes for the same. Suppose they started at the same point and at the same time and went in the same direction.

After how many minutes have they met again at the starting point? 4

❐❐❐

CHAPTER

2 Polynomials

KEY POINTS

1. **Polynomial :** If x is a variable, n is a natural number and $a_0, a_1, a_2, a_3, \ldots\ldots\ldots a_n$ are real numbers, then $p(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots\ldots + a_1 x + a_0$, $(a_n \neq 0)$ is called a polynomial in x.
2. Polynomials of degree 1, 2 and 3 are called linear, quadratic and cubic polynomials respectively.
3. A quadratic polynomial is an algebraic expression of the form $ax^2 + bx + c$, where a, b, c are real numbers with $a \neq 0$.
4. Zeros of a polynomial $p(x)$ are precisely the x – coordinates of the points where the graph of $y = p(x)$ intersects the x–axis, *i.e.,* $x = a$ is a zero of polynomial $p(x)$ if $p(a) = 0$
5. A polynomial can have at most the same number of zeros as the degree of the polynomial.
6. (*i*) If one zero of a quadratic polynomial $p(x)$ is negative of the other, then coefficient of x is 0.
   (*ii*) If zeroes of a quadratic polynomial $p(x)$ are reciprocal of each other, then coefficient of x^2 = constant term.
7. Relationship between zeros and coefficients of a polynomial
   If α and β are zeros of $p(x) = ax^2 + bx + c$ $(a \neq 0)$, then
   Sum of zeros = $\alpha + \beta = -\frac{b}{a}$
   Product of zeros = $\alpha\beta = \frac{c}{a}$
8. If α, β are zeros of a quadratic polynomial $p(x)$, then
   $p(x) = k\,[x^2 - (\text{sum of zeros})\,x + \text{product of zeros}]$
   $\Rightarrow\ p(x) = k\,[x^2 - (\alpha + \beta)x + \alpha\beta\,]$; where k is any non-zero real number.
9. Graph of linear polynomial $p(x) = ax + b$ is a straight line.
10. Division Algorithm states that given any polynomials $p(x)$ and $g(x)$, there exist polynomial $q(x)$ and $r(x)$ such that:

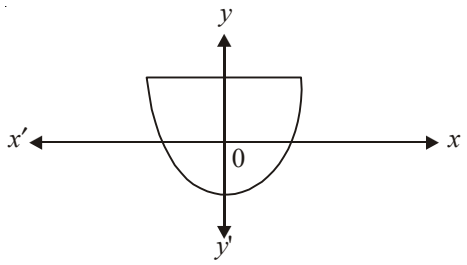
$p(x) = g(x).\ q(x) + r(x)\ ;\ g(x) \neq 0,$

[where either $r(x) = 0$ or degree $r(x) <$ degree $g(x)$]

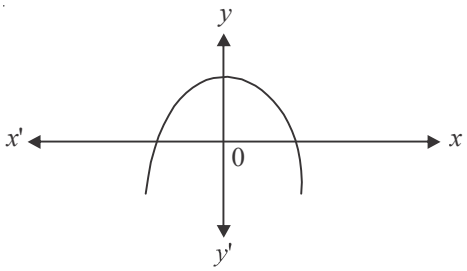
Graph of different types of polynomials:

- **Linear Polynomial :** The graph of a linear polynomial $ax + b$ is a straight line, intersecting x-axis at one point.
- **Quadratic Polynomial:**

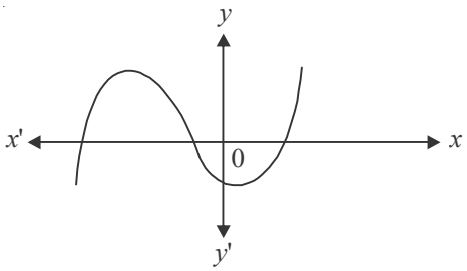
(*i*) Graph of a quadratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open upwards like U, if a > 0 and intersect x-axis at maximum two distinct points.



(*ii*) Graph of a quodratic polynomial $p(x) = ax^2 + bx + c$ is a parabola open downwards like $\cap$, if a < 0 and intersect x-axis at maximum two distinct points.



(*iii*) Polynomial and its graph : In general a polynomial $p(x)$ of degree n crosses the x-axis at most n points.



VERY SHORT ANSWER TYPE QUESTIONS

1. If one root of the polynomial $P(x) = 5x^2 + 13x + K$ is reciprocal of the other, then value of k is

(a) 0 (b) 5 (c) $\frac{1}{6}$ (d) 6

2. If α and β are the zeroes of the polynomial $p(x) = x^2 - p(x + 1) - c$ such that $(\alpha + 1)(\beta + 1) = 0$, the c = _____ .

3. If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(a) 10 (b) – 10 (c) 5 (d) – 5

4. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and – 3, then

(a) $a = -7, b = -1$ (b) $a = 5, b = -1$

(c) $a = 2, b = -6$ (d) $a = 0, b = -6$

5. What should be added to the polynomial $x^2 - 5x + 4$, so that 3 is the zero of the resulting polynomial:

(a) 1 (b) 2 (c) 4 (d) 5

6. If α and β are the roots of the polynomial

$f(x) = x^2 + x + 1$, then $\frac{1}{\alpha} + \frac{1}{\beta} =$

7. If a quadratic polynomial $f(x)$ is not factorizable into linear factors, then it has no real zero. (True/False)

8. If a quadratic polynomial $f(x)$ is a square of a linear polynomial, then its two zeros are coincident. (True/False).

9. The product of the zeros of $x^3 + 4x^2 + x - 6$ is

(a) – 4 (b) 4 (c) 6 (d) 6

10. Given that two of the zeros of the cubic polynomial $ax^3 + bx^2 + cx + d$ are 0, the third zero is

(a) $-\frac{b}{a}$ (b) $\frac{b}{a}$ (c) $\frac{c}{a}$ (d) $-\frac{d}{a}$

11. What will be the number of zeros of a linear polynomial $p(x)$ if its graph (*i*) passes through the origin. (*ii*) doesn't intersect or touch x-axis at any point?

12. Find the quadratic polynomial whose zeros are

$\left(5 + 2\sqrt{3}\right)$ and $\left(5 - 2\sqrt{3}\right)$

13. If one zero of $p(x) = 4x^2 - (8k^2 - 40k)\,x - 9$ is negative of the other, find values of k.

14. What number should be added to the polynomial $x^2 - 5x + 4$, so that 3 is a zero of polynomial so obtained.

15. How many (*i*) maximum (*ii*) minimum number of zeroes can a quadratic polynomial have?

16. What will be the number of real zeros of the polynomial $x^2 + 1$?

17. If α and β are zeros of polynomial $6x^2 - 7x - 3$, then form a quadratic polynomial where zeros are 2α and 2β (CBSE)

18. If α and $\frac{1}{\alpha}$ are zeros of $4x^2 - 17x + k - 4$, find the value of k.

19. What will be the number of zeros of the polynomials whose graphs are parallel to (*i*) y-axis (ii) x-axis?

20. What will be number of zeros of the polynomials whose graphs are either touching or intersecting the axis only at the points:

(*i*) $(-3, 0)$, $(0, 2)$ & $(3, 0)$ (*ii*) $(0, 4)$, $(0, 0)$ and $(0, -4)$

SHORT ANSWER TYPE (I) QUESTIONS

21. If -3 is one of the zeros of the polynomial $(k - 1)x^2 + k\,x + 1$, find the value of k.

22. If the product of zeros of $ax^2 - 6x - 6$ is 4, find the value of a. Hence find the sum of its zeros.

23. If zeros of $x^2 - kx + 6$ are in the ratio 3 : 2, find k.

24. If one zero of the quadratic polynomial $(k^2 + k)x^2 + 68x + 6k$ is reciprocal of the other, find k.

25. If α and β are the zeros of the polynomial $x^2 - 5x + m$ such that $\alpha - \beta = 1$, find m. (CBSE)

26. If the sum of squares of zeros of the polynomial $x^2 - 8x + k$ is 40, find the value of k.

27. If α and β are zeros of the polynomial $t^2 - t - 4$, form a quadratic polynomial whose zeros are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

28. What should be added to the polynomial $x^3 - 3x^2 + 6x - 15$, so that it is completely divisible by $x - 3$? (CBSE 2016)

29. If m and n are the zeros of the polynomial $3x^2 + 11x - 4$, find the value of $\frac{m}{n} + \frac{n}{m}$.

(CBSE, 2012)

30. Find a quadratic polynomial whose zeros are $\frac{3+\sqrt{5}}{5}$ and $\frac{3-\sqrt{5}}{5}$.

(CBSE, 2013)

SHORT ANSWER TYPE (II) QUESTIONS

31. If $(k + y)$ is a factor of each of the polynomials $y^2 + 2y - 15$ and $y^3 + a$, find the values of k and a.

32. Obtain zeros of $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3}$ and verify relation between its zeroes and coefficients.

33. If $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by $(x^2 + 5)$, remainder comes out to be $(px + q)$, find values of p and q.

34. -5 is one of the zeros of $2x^2 + px - 15$, zeroes of $p(x^2 + x) + k$ are equal to each other. Find the value of k.

35. Find the value of k such that $3x^2 + 2kx + x - k - 5$ has the sum of zeros as half of their product.

36. If α and β are zeros of $y^2 + 5y + m$, find the value of m such that $(\alpha + \beta)^2 - \alpha\beta = 24$

37. If α and β are zeros of $x^2 - x - 2$, find a polynomial whose zeros are $(2\alpha + 1)$ and $(2\beta + 1)$

38. Find values of a and b so that $x^4 + x^3 + 8x^2 + ax + b$ is divisible by $x^2 + 1$.

39. What must be subtracted from $8x^4 + 14x^3 - 2x^2 + 7x - 8$ so that the resulting polynomial is exactly divisible by $4x^2 + 3x - 2$?

40. What must be added to $4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is divisible by $x^2 - 2x - 3$?

LONG ANSWER TYPE QUESTIONS

41. Find all zeros of the polynomial $2x^3 + x^2 - 6x - 3$ if two of its zeroes are $\sqrt{3}$ and $-\sqrt{3}$.

42. If $\sqrt{2}$ is a zero of $(6x^3 + \sqrt{2}\,x^2 - 10x - 4\sqrt{2})$, find its other zeroes.

43. If two zeros of $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $(2 \pm \sqrt{3})$, find other zeroes.

44. On dividing the polynomial $x^3 - 5x^2 + 6x - 4$ by a polynomial $g(x)$, quotient and remainder are $(x - 3)$ and $(-3x + 5)$ respectively. Find $g(x)$

45. Obtain all zeros of the polynomial $2x^4 - 2x^3 - 7x^2 + 3x + 6$ if two factors of this polynomial are $\left(x \pm \sqrt{\frac{3}{2}}\right)$.

46. If the polynomial $x^4 - 3x^3 - 6x^2 + kx - 16$ is exactly divisible by $x^2 - 3x + 2$, then find the value of k. (CBSE, 2014)

47. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by $x^2 - 2x + k$, then find the vlaue of k and a. (CBSE)

48. If α and β are zeros of the polynomial $x^2 + 4x + 3$, find the polynomial whose zeros are $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$. (CBSE)

49. Find K, so that $x^2 + 2x + K$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeros of the two polynomials: (Exempler, HOTS)

50. If $x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, then find all the zeros of the polynomial.

ANSWERS AND HINTS

1. (b) 5

2. –1

3. (b) –10

4. (d) a = 0, b = –6

5. (b) 2

6. – 1

7. True

8. True

9. (c) 6

10. (a) $-\frac{b}{a}$

11. (i) 1 (ii) 0

12. $x^2 - 10x + 13$

13. $k = 0, 5$

14. 2

15. (i) 2 (ii) 0

16. 0

17. $3x^2 - 7x - 6$

18. $k = 8$

19. (i) 1 (ii) 0

20. (i) 2 (ii) 1

21. 4/3

22. $a = -\frac{3}{2}$, sum of zeroes $= -4$

23. $-5, 5$

24. 5

25. 6

26. 12

27. $4t^2 + t - 1$

28. On dividing $x^3 - 3x^2 + 6x - 15$ by x – 3, remainder is + 3, hence – 3 must be added to $x^3 - 3x^2 + 6x - 15$.

29. $\frac{m}{n} + \frac{n}{m} = \frac{m^2 + n^2}{mn} = \frac{(m+n)^2 - 2mn}{mn} = \frac{\left(-\frac{11}{3}\right)^2 - 2\left(-\frac{4}{3}\right)}{-\frac{4}{3}} = -\frac{145}{12}$

30. $\alpha + \beta = \frac{6}{5}, \quad \alpha\beta = \frac{4}{25}$,
$25x^2 - 30x + 4$

31. $k = 3, -5$ and $a = 27, -125$

32. $-\frac{2}{\sqrt{3}}, \quad \frac{\sqrt{3}}{4}$

33. $p = 2, \ q = 3$

34. $\frac{7}{4}$

35. 1

36. 1

37. $x^2 - 4x - 5$

38. $a = 1, b = 7$

39. $14x - 10$

40. $61x - 65$

41. $\sqrt{3}, \ -\sqrt{3}, \ -\frac{1}{2}$

42. $-\frac{\sqrt{2}}{2}, \ \frac{-2\sqrt{2}}{3}$

43. $-5, \ 7$

44. $x^2 - 2x + 3$ **45.** $2, \ -1, \ \mp\sqrt{\frac{3}{2}}$

46. $x^2 - 3x + 2 = (x - 2)(x - 1)$

$P(1) = 0, K = 24.$

47. On dividing $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$ we get remainder $(2k - 9)x + (10 - 8k + k^2)$

Given remainder $= x + 9$

$$2k - 9 = 1 \quad \Rightarrow \quad k = 5$$

$$10 - 8k + k^2 = a \quad \Rightarrow \quad a = 10 - 40 + 25 = -5$$

$$a = -5, \quad k = 5$$

48. $x^2 - \frac{16}{3}x + \frac{16}{3}$ or $\frac{1}{3}(3x^2 - 16x + 16)$

49. On dividing $2x^4 + x^3 - 14x^2 + 5x + 6$ by $x^2 + 2x + k$

We get $(7k + 21)x + 2k^2 + 8k + 6$ as remainder is zero.

$\Rightarrow$ $7k + 21 = 0$ and $2k^2 + 8k + 6 = 0$

$\Rightarrow$ $k = -3$ and $k = -1$ or -3

$\Rightarrow$ $k = -3$

Zeros of $x^2 + 2x - 3$ are $1, -3$ and $2x^4 + x^3 - 14x^2 + 5x + 6$ are $1, \ -3, \ 2, \ -\frac{1}{2}$

50. $\sqrt{5}, \ \sqrt{5} + \sqrt{2}, \ \sqrt{5} - \sqrt{2}$

PRACTICE-TEST

Polynomials

Time : 1 Hr. ***M.M. : 20***

SECTION-A

1. If α and β are zeros of a quadratic polynomial $p(x)$, then factorize $p(x)$. **1**

2. If α and β are zeros of $x^2 - x - 1$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$. **1**

3. If one of the zeros of quadratic polynomial $(K-1)x^2 + kx + 1$ is -3 then the value of K is, **1**

(a) $\frac{4}{3}$ (b) $-\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

4. A quadratic polynomial, whose zeros are -3 and 4, is **1**

(*a*) $x^2 - x + 12$ (b) $x^2 + x + 12$

(*c*) $\frac{x^2}{2} - \frac{x}{2} - 6$ (d) $2x^2 + 2x - 24$

SECTION-B

5. If α and β are zeros of $x^2 - (k+6)x + 2(2k-1)$. find the value of k if $\alpha + \beta = \frac{1}{2}\alpha\beta$. **2**

6. Find a quadratic polynomial one of whose zeros is $(3+\sqrt{2})$ and the sum of its zeroes is 6. **2**

7. If zeros of the polynomial $x^2 + 4x + 2a$ are α and $\frac{2}{\alpha}$ then find the value of a. **2**

SECTION-C

8. Find values of a and b if $(x^2 + 1)$ is a factor of the polynomial $x^4 + x^3 + 8x^2 + ax + b$. **3**

9. If truth and lie are zeros of the polynomial $px^2 + qx + r$, $(p \neq 0)$ and zeros are reciprocal to each other, Find the relation between p and r. **3**

SECTION-D

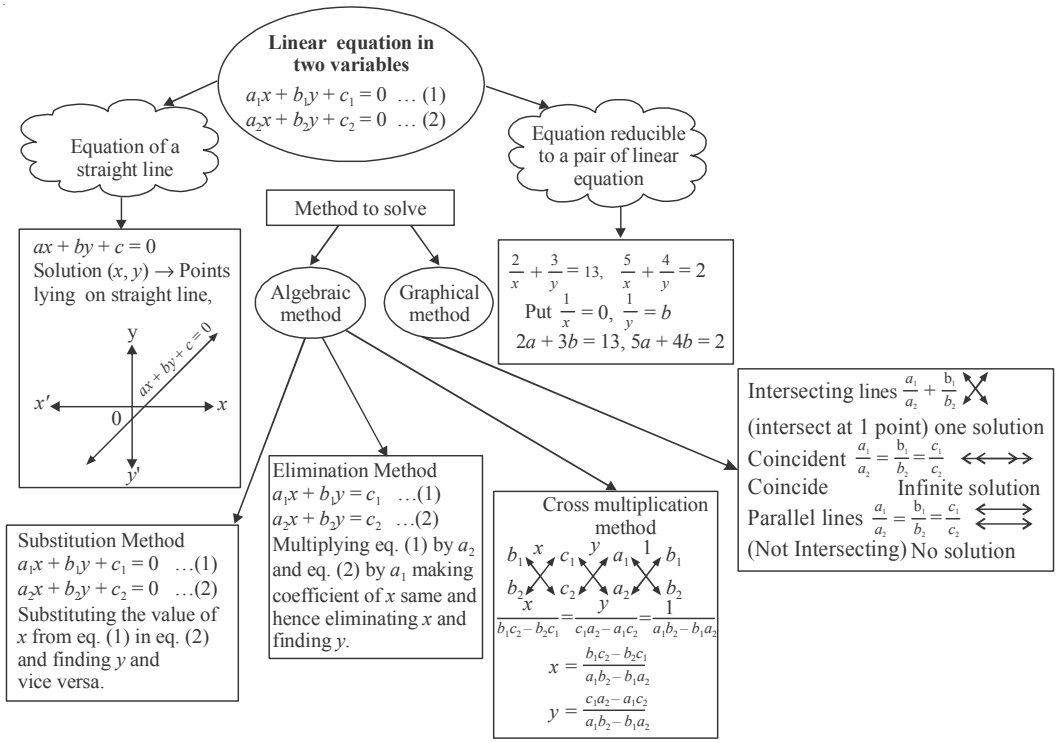
10. On dividing the polynomial $x^3 + 2x^2 + kx + 7$ by $(x - 3)$, remainder comes out to be 25. Find quotient and the value of k. Also find the sum and product of zeros of the quotient so obtained. **4**

□□□

CHAPTER

3 Pair of Linear Equations in Two Variables

KEY POINTS



VERY SHORT ANSWER TYPE QUESTIONS

1. If the lines given by $3x + 2ky = 2$ and $2x + 5y = 1$ are parallel, then the value of k is _____ .

2. If $x = a$ and $y = b$ is the solution of the equation $x - y = 2$ and $x + y = 4$, then the values of a and b are respectively _____ .

3. A pair of linear equations which has a unique solution $x = 2$ and $y = -3$ is

(a) $x + y = 1$ and $2x - 3y = -5$

(b) $2x + 5y = -11$ and $2x - 3y = -22$

(c) $2x + 5y = -11$ and $4x + 10y = 22$

(d) $x - 4y - 14 = 0$ and $5x - y - 13 = 0$

4. The area of the triangle formed by the lines $x = 3$, $y = 4$ and $x = y$ is _____ .

5. The value of K for which the system of equations $3x + 5y = 0$ and $kx + 10y = 0$ has a non-zero solutions is ____ .

6. If a pair of linear equations in two variables is consistent, then the lines represented by two equations are:

(a) Intersecting (b) Parallel

(c) always coincident (d)intersecting or coincident

7. For $2x + 3y = 4$, y can be written in terms of x as _____ .

8. One of the common solution of $ax + by = c$ and y axis is

(a) $\left(0, \frac{c}{b}\right)$ (b) $\left(0, \frac{b}{c}\right)$

(c) $\left(\frac{c}{b}, 0\right)$ (d) $\left(0, -\frac{c}{b}\right)$

9. If $ax + by = c$ and $lx + my = n$ has unique solution then the relation between the coefficient will be:

(a) $am \neq lb$ (b) $am = lb$ (c) $ab = lm$ (d) $ab \neq lm$

10. In ΔABC, $\angle C = 3\angle B$, $\angle C = 2(\angle A + \angle B)$ then, $\angle A$, $\angle B$, $\angle C$ are respectively.

(a) 30°, 60°, 90° (b) 20°, 40°, 120°

(c) 45°, 45°, 90° (d) 110°, 40°, 50°

11. If $x = 3m - 1$ and $y = 4$ is a solution of the equation $x + y = 6$, then find the value of m.

12. What is the point of intersection of the line represented by $3x - 2y = 6$ and the y-axis?

13. For what value of p, system of equations $2x + py = 8$ and $x + y = 6$ have no solution.

14. A motor cyclist is moving along the line $x - y = 2$ and another motor cyclist is moving along the line $x - y = 4$ find out their moving direction.

15. Find the value of k for which pair of linear equations $3x + 2y = -5$ and $x - ky =$

2 has a unique solution.

16. Express y in terms of x in the expression $3x - 7y = 10$

17. If $2x + 5y = 4$, write another linear equation, so that lines represented by the pair are coincident.

18. Check whether the graph of the pair of linear equations $x + 2y - 4 = 0$ and $2x + 4y - 12 = 0$ is intersecting lines or parallel lines.

19. If the lines $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then find value of k.

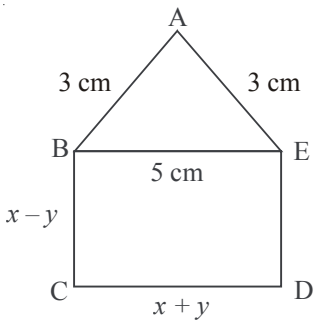
20. If we draw lines of $x = 2$ and $y = 3$ what kind of lines do we get?

SHORT ANSWER TYPE (I) QUESTIONS (2 MARKS QUESTIONS)

21. Form a pair of linear equations for: The sum of the numerator and denominator of the fraction is 3 less than twice the denominator. If the numerator and denominator both are decreased by 1, the numerator becomes half the denominator.

22. For what value of p the pair of linear equations $(p + 2)x - (2p + 1)y = 3(2p - 1)$ and $2x - 3y = 7$ has a unique solution.

23. ABCDE is a pentagon with BE || CD and BC || DE, BC is perpendicular to CD If the perimeter of ABCDE is 21 cm, find x and y.



24. Solve for x and y

$$x - \frac{y}{2} = 3 \text{ and } \frac{x}{2} - \frac{2y}{3} = \frac{2}{3}$$

25. Solve for x and y

$3x + 2y = 11$ and $2x + 3y = 4$

Also find p if $p = 8x + 5y$

26. Solve the pair of linear equations by substitution method $x - 7y + 42 = 0$ and $x - 3y - 6 = 0$

27. Ram is walking along the line joining (1, 4) and (0, 6)

Rahim is walking along the line Joining (3, 4) and (1, 0)

Represent on graph and find the point where both of them cross each other

28. Given the linear equation $2x + 3y - 12 = 0$, write another linear equation in these variables, such that. geometrical representation of the pair so formed is

(*i*) Parallel Lines (*ii*) Coincident Lines

29. The difference of two numbers is 66. If one number is four times the other, find the numbers.

30. For what value of k, the following system of equations will be inconsistent

$$kx + 3y = k - 3$$

$$12x + ky = k$$

SHORT ANSWERS TYPE (II) QUESTIONS

31. Solve graphically the pair of linear equations $5x - y = 5$ and $3x - 2y = -4$

Also find the co-ordinates of the points where these lines intersect y-axis

32. Solve for x and y

$$\frac{5}{x+y} + \frac{1}{x-y} = 2$$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2$$

33. Solve by Cross – multiplication method (CBSE)

$$\frac{x}{a} + \frac{y}{b} = a + b$$

$$\frac{x}{a^2} + \frac{y}{b^2} = 2$$

34. For what values of a and b the following pair of linear equations have infinite number of solutions? (CBSE)

$$2x + 3y = 7$$

$$a(x + y) - b(x - y) = 3a + b - 2$$

35. Solve the pair of linear equations

$$152x - 378y = -74$$
$$-378x + 152y = -604$$

36. Pinky scored 40 marks in a test getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks were deducted for each wrong answer, then pinky again would have scored 40 marks. How many questions were there in the test?

37. A two digit number is obtained by either multiplying sum of digits by 8 and adding 1 or by multiplying the difference of digits by 13 and adding 2. Find the number

38. Father's age is three times the sum of ages of his two children. After 5 years his age will be twice the sum of ages of two children. Find the age of the father.

39. On selling a T.V. at 5% gain and a fridge at 10% gain, a shopkeeper gain ₹ 2000. But if he sells the T.V. at 10% gain and fridge at 5% loss, he gains ₹ 1500 on the transaction. Find the actual price of the T.V. and the fridge

40. Sunita has some ₹ 50 and ₹ 100 notes amounting to a total of ₹ 15,500. If the total number of notes is 200, then find how many notes of ₹ 50 and ₹ 100 each, she has.

LONG ANSWER TYPE QUESTIONS

41. Solve graphically the pair of linear equations $3x - 4y + 3 = 0$ and $3x + 4y - 21 = 0$ Find the co-ordinates of vertices of triangular region formed by these lines and x-axis. Also calculate the area of this triangle.

42. Solve for x and y

$$\frac{1}{2(2x+3y)} + \frac{12}{7(3x-2y)} = \frac{1}{2}$$

$$\frac{7}{(2x+3y)} + \frac{4}{(3x-2y} = 2$$

For $2x + 3y \neq 0$

$3x - 2y \neq 0$.

43. Solve the pair of equations by reducing them to a pair of linear equations

$$\frac{3x+2y}{xy} = 1 \text{ and } \frac{4x-2y}{xy} = 13$$

hence find a for which $y = ax - 4$

44. A man travels 600 km to his home partly by train and partly by bus. He takes 8

hours, if he travels 120 km by train and rest by bus. Further, it takes 20 minute longer, if he travels 200 km by train and rest by bus. Find the speeds of the train and the bus.

45. A and B are two points 150 km apart on a highway. Two cars start with different speeds from A and B at same time. If they move in same direction, they meet in 15 hours. If they move in opposite direction, they meet in one hour. Find their speeds

46. A boat covers 32 km upstream and 36 km downstream, in 7 hours. Also it Covers 40 km upstream and 48 km downstream in 9 hours. Find the speed of boat in still water and that of the stream. (CBSE)

47. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction.

48. 8 Women and 12 men can complete a work in 10 days while 6 women and 8 men can complete the same work in 14 days. Find the time taken by one woman alone and that one man alone to finish the work.

49. The ratio of incomes of two persons A and B is 3 : 4 and the ratio of their expenditures is 5 : 7. If their savings are ₹ 15,000 annually find their annual incomes.

50. Vijay had some bananas and he divided them into two lots A and B. He sold the first lot at the rate of ₹ 2 for 3 bananas and the second lot at the rate of ₹ 1 per banana and got a total of ₹ 400. If he had sold the first lot at the rate of ₹ 1 per banana and the second lot at the rate of ₹ 4 for 5 bananas, his total collection would have been ₹ 460. Find the total number of bananas he had.

(HOTS, Exampler)

51. A railway half ticket cost half the full fare but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket costs ₹ 2530. One reserved first class ticket and one reserved first class half ticket from stations A to B costs ₹ 3810. Find the full first class fare from stations A to B and also the reservation charges for a ticket. (Exemplar)

52. Solve the following pair of equations.

$\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$ and $\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$ (CBSE, 2015)

53. Determine graphically, the vertices of the triangle formed by the times $y = x$, $3y = x$ and $x + y = 8$. (NCERT Exemplar).

54. Draw the graphs of the equations x = 3, x = 5 and $2x - y - 4 = 0$. Also find the area of the quadrilateral formed by the lines and the x-axis.
(NCERT Exemplar, HOTS)

55. The area of a rectangle gets reduced by a 9 square units, if its length is reduced by 5 units and the breadth is increased by 3 units. The area is increased by 67 sqaure units if length is increased by 3 units and breadth is increased by 2 units. Find the perimeter of the rectangle. (CBSE)

ANSWERS AND HINTS

1. $K = \frac{15}{4}$ **2.** $a = 3$ and $b = 1$

3. (*b*) 2x + 5y = – 11 and $4x + 10y = -22$

4. $\frac{1}{2}$ sq. unit **5.** 6

6. (*d*) intersecting or coincident **7.** $y = \frac{4 - 2x}{3}$

8. (*a*) $\left(0, \frac{c}{b}\right)$ **9.** (*a*) am ≠ lb

10. (*b*) 20°, 40°, 120° **11.** $m = 1$

12. (0, –3) **13.** $p = 2$

14. move parallel **15.** $k \neq \frac{-2}{3}$

16. $y = \frac{3x - 10}{7}$ **17.** $4x + 10y = 8$

18. Parallel lines 1**9.** $k = \frac{15}{4}$

20. Intersecting lines **21.** $x - y = -3$, $2x - y = 1$

12. $p \neq 4$ **23.** $x = 5, y = 0$

24. 4, 2 **25.** $x = 5, y = -2, p = 30$

26. 42, 12 **27.** (2, 2)

28. (*i*) $4x + 6y + 10 = 0$
(*ii*) $4x + 6y - 24 = 0$

29. 88, 22 **30.** $k = -6$

31. (2, 5) (0, – 5) and (0, 2) **32.** (3, 2)

33. $x = a^2, y = b^2$ **34.** $a = 5, b = 1$

35. 2, 1 **36.** 40 questions

37. 41 **38.** 45 years

39. T.V. = ₹ 20,000 Fridge = ₹ 10,000

40. ₹ 50 notes = 90, ₹ 100 notes = 110

41. Solution (3, 3), Vertices (– 1, 0) (7, 0) and (3, 3), Area = 12 square unit

42. (2, 1) **43.** $x = \frac{-2}{5}, y = \frac{1}{2}, a = \frac{-45}{4}$

44. 60 km/hr, 80 km/hr **45.** 80 km/hr , 70 km/hr

46. 10 km/hr, 2 km/hr **47.** $\frac{5}{9}$

48. 1 woman in 140 days, 1 man in 280 days

49. ₹ 90,000, ₹ 1,20,000

50. Let the no. of bananas in lots A be x and in lots B be y

Case I : $\frac{2}{3}x + y = 400 \quad \Rightarrow \quad 2x + 3y = 1200$

Case 2 : $x + \frac{4}{5}y = 460 \quad \Rightarrow \quad 5x + 4y = 2300$

$x = 300, \quad y = 200$, Total bananas = 500.

51. Let the cost of full and half ticket be ₹ x & ₹ $\frac{x}{2}$ and reservation charge by ₹ y per ticket.

Case I : $x + y = 2530$

Case 2 : $x + y + \frac{x}{2} + y = 3810$

$x = 2500, y = 3810$

Full first class fare is ₹ 2500 and reservation charge is ₹ 30.

52. $x = 4, y = 9$

53. Vertices of the triangle are (0, 0) (4, 4) (6, 2).

54. Area of quadrilateral ABCD where,

$A(3, 0), B(5, 0)$

C(5, 6), $D(3, 2)$

$$= \frac{1}{2} \times AB \times (AD + BC)$$

$$= \frac{1}{2} \times 2 \times (6 + 2) = 8 \text{ sq. units.}$$

55. Length of rectangle is 17 units.

Breadth of rectangle is 9 units.

Perimeter of rectangle is 52 units.

❐❐❐

PRACTICE-TEST

Pair of Linear Equations In Two Variables

Time : 1 Hr. ***M.M. : 20***

SECTION-A

1. For what value of k system of equations
$x + 2y = 3$ and $5x + ky + 7 = 0$ has a unique solution. **1**

2. Does the point (2, 3) lie on line of graph of $3x - 2y = 5$. **1**

3. The pair of equations $x = a$ and $y = b$ graphically representes lines which are: **1**

(a) Parallel (b) Intersecting at (b, a)

(c) Coincident (d) Intersecting at (a, b)

4. For what value of K, do the equation $3x - y + 8 = 0$ and $6x - Ky = -16$ represent coincident lives? **1**

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 2 (d) –2

SECTION-B

5. For what values of a and b does the pair of linear equations have infinite number of solutions

$$2x - 3y = 7$$
$$ax + 3y = b$$ 2

6. Solve for x and y

$$0.4x + 0.3y = 1.7$$
$$0.7x - 0.2y = 0.8$$ **2**

7. If the system of equations $6x + 2y = 3$ and $kx + y = 2$ has a unique solution, find the value of k. **2**

SECTION-C

8. Solve for x and y by cross multiplication method

$$x + y = a + b$$
$$ax - by = a^2 - b^2$$ **3**

9. Sum of the ages of a father and the son is 40 years. If father's age is three times that of his son, then find their ages. **3**

SECTION-D

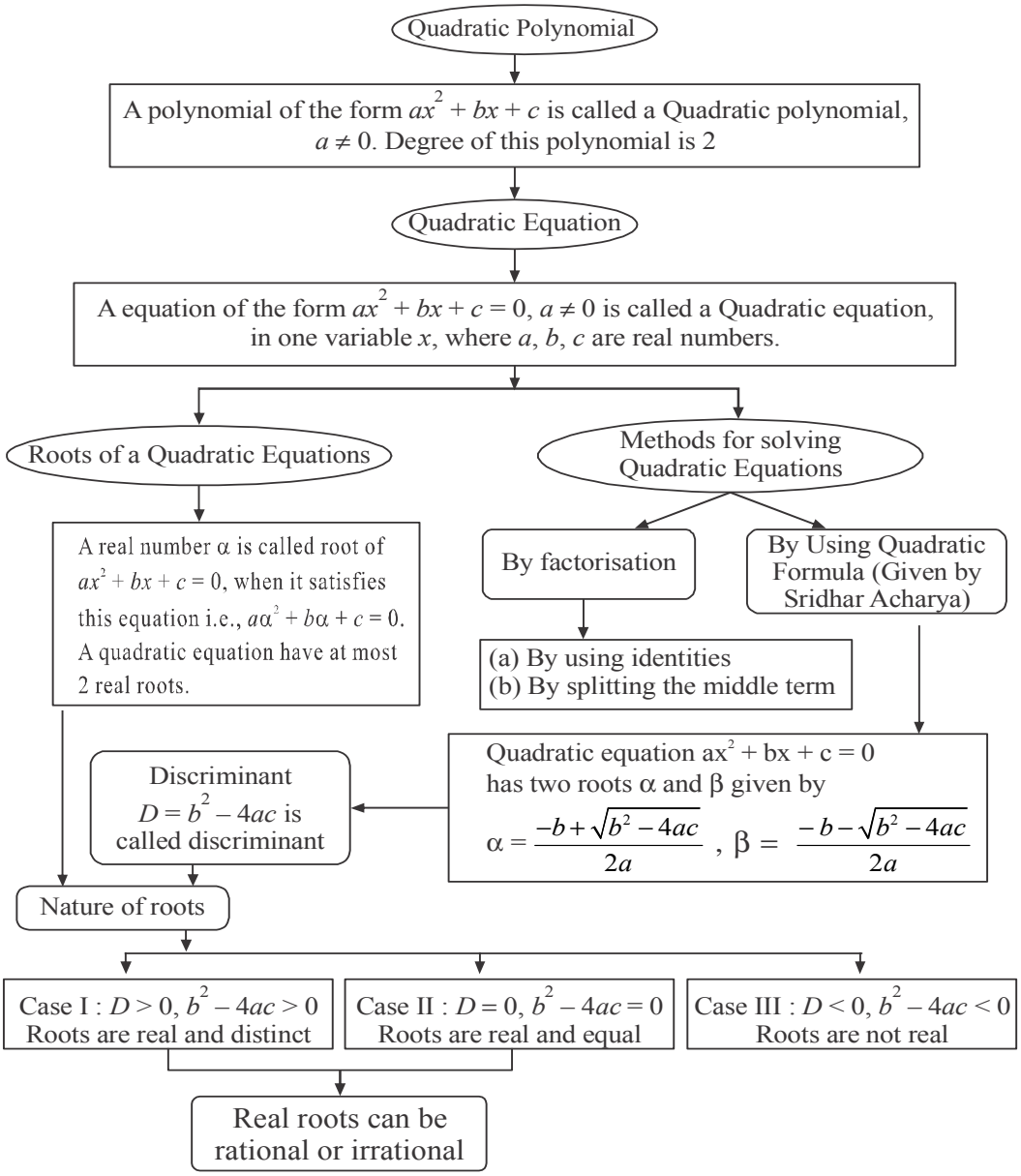
10. Solve the following pair of equations graphically.
$3x + 5y = 12$ and $3x - 5y = -18$. **4**

Also shade the region enclosed by these two lines and x-axis. ❐❐❐

CHAPTER

4 Quadratic Equations

Basic Concepts



NOTES:

1. Real and distinct roots are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
2. Real and equal roots are $\frac{-b}{2a}, \frac{-b}{2a}$
3. There are quadratic equation which donot have any real roots e.g. $x^2 + 1 = 0$

VERY SHORT ANSWER TYPE QUESTIONS

Multiple Choice Questions:

1. Which of the following is not a Quadratic Equation?

(*a*) $2(x - 1)^2 = 4x^2 - 2x + 1$ (*b*) $3x - x^2 = x^2 + 6$

(*c*) $\left(\sqrt{3}x + \sqrt{2}\right)^2 = 2x^2 - 5x$ (*d*) $(x^2 + 2x)^2 = x^4 + 3 + 4x^2$

2. Which of the following equation has 2 as a root

(*a*) $x^2 + 4 = 0$ (*b*) $x^2 - 4 = 0$

(*c*) $x^2 + 3x - 12 = 0$ (*d*) $3x^2 - 6x - 2 = 0$

3. If $\frac{1}{2}$ is a root of $x^2 + px - \frac{5}{4} = 0$ then value of p is

(*a*) 2 (*b*) –2

(*c*) $\frac{1}{4}$ (*d*) $\frac{1}{2}$

4. Every Quadratic Equation can have at most

(*a*) Three roots (*b*) One root

(*c*) Two roots (*d*) Any number of roots

5. Roots of Quadratic equation $x^2 - 7x = 0$ will be

(*a*) 7 (*b*) 0, –7

(*c*) 0, 5 (*d*) 0, 7

6. Fill in the blanks:

(*a*) If $px^2 + qx + r = 0$ has equal roots then value of r will be _____ .

(*b*) The qaudratic equation $x^2 - 5x - 6 = 0$ if expressed as $(x + p)(x + q) = 0$ then value of p and q respectively are _____ and _____ .

(*c*) The value of k for which the roots of qaudratic equations $x^2 + 4x + k = 0$ are real is _____ .

(*d*) If roots of $4x^2 - 2x + c = 0$ are reciprocal of each other then the value of c is _____ .

(*e*) If in a quadratic equation $ax^2 + bx + c = 0$, value of a is zero then it become a _____ equation.

7. Write whether the following statements are true or false. Justify your answers.

(*a*) Every quadratic equation has atleast one real roots.

(*b*) If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.

(*c*) 0.3 is a root of $x^2 - 0.9 = 0$.

(*d*) The graph of a quadratic polynomial is a straight line.

(*e*) The discriminant of $(x - 2)^2 = 0$ is positive.

8. Match the following :

(*i*) Roots of $3x^2 - 27 = 0$	(*a*) 169/9
(*ii*) D of $2x^2 + \frac{5}{3}x - 2 = 0$	(*b*) 0
(*iii*) Sum of roots of $8x^2 + 2x - 3 = 0$	(*c*) $x^2 - (a + b)x + ab = 0$
(*iv*) A quadratic equation with roots a and b	(*d*) 3, – 3
(*v*) The product of roots of $x^2 + 8x = 0$	(*e*) $\frac{-1}{4}$

SHORT ANSWER TYPE QUESTIONS-I

9. If the Quadratic equation $Px^2 - 2\sqrt{5}\,Px + 15 = 0$ has two equal roots then find the value of P.

10. Solve for x by factorisation

(*a*) $8x^2 - 22x - 21 = 0$

(*b*) $3\sqrt{5}x^2 + 25x + 10\sqrt{5} = 0$

(*c*) $3x^2 - 2\sqrt{6}\,x + 2 = 0$ **(CBSE 2010)**

(*d*) $2x^2 - ax + a^2 = 0$ **(CBSE 2014)**

(*e*) $\sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$

(*f*) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(*g*) $(x - 1)^2 - 5(x - 1) - 6 = 0$

11. If – 5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quandratic equation $p(x^2 + x) + k = 0$ has equal roots find the value of k. **(CBSE 2014, 2016)**

12. If $x = \frac{2}{3}$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$. Find the value of a and b. **(CBSE 2016)**

13. Find value of p for which the product of roots of the quadratic equation $px^2 + 6x + 4p = 0$ is equal to the sum of the roots.

14. The sides of two squares are x cm and $(x + 4)$ cm. The sum of their areas is 656 cm^2 Find the sides of these two squares.

15. Find K if the difference of roots of the quadratic equation $x^2 - 5x + (3k - 3) = 0$ is 11.

SHORT ANSWER TYPE QUESTIONS-II

16. Find the positive value of k for which the quadratic equation $x^2 + kx + 64 = 0$ and the quadratic equation $x^2 - 8x + k = 0$ both will have real roots.

17. Solve for x

(a) $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$ $a + b + x \neq 0$, $a, b, x \neq 0$ **(CBSE 2005)**

(b) $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$ $2a + b + 2x \neq 0$, $a, b, x \neq 0$

(c) $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0, x \neq 3, \frac{-3}{2}$

(e) $\frac{1}{x-1} - \frac{1}{x+5} = \frac{6}{7}, x \neq 1, 5$ **(CBSE 2010)**

(d) $4x^2 + 4bx - (a^2 - b^2) = 0$

(f) $4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$

(g) $\frac{2}{x+1} + \frac{3}{2(x-2)} = \frac{23}{5x}, x \neq 0, -1, 2$

(h) $\left(\frac{2x}{x-5}\right)^2 + \frac{10x}{(x-5)} - 24 = 0, x \neq 5$

(i) $4x^2 - 4a^2x + a^4 - b^4 = 0$

(j) $2a^2x^2 + b(6a^2 + 1)x + 3b^2 = 0$

(k) $3\left(\frac{7x+1}{5x-3}\right) - 4\left(\frac{5x-3}{7x+1}\right) = 11, x \neq \frac{3}{5}, \frac{-1}{7}$

(*l*) $\frac{1}{x+4}-\frac{1}{x-7}=\frac{11}{30}, x \neq -4, 7$ **(NCERT)**

(*m*) $\frac{x-4}{x-5}+\frac{x-6}{x-7}=\frac{10}{3}, x \neq 5, 7$ **(CBSE 2014)**

(*n*) $\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4}, \quad x \neq -1, -2, -4$

(*o*) $\frac{1}{2x-3}+\frac{1}{x-5}=1, \quad x \neq \frac{3}{2}, 5$

(*p*) $x^2 + 5\sqrt{5}x - 70 = 0$

(*q*) $\frac{16}{x}-1=\frac{15}{x+1}, x \neq 0, -1$ **(CBSE 2014)**

18. Solve by using quadratic formula $abx^2 + (b^2 - ac)x - bc = 0$. **(CBSE 2005)**

19. If the roots of the quandratic equation $(p+1)x^2 - 6(p+1)x + 3(p+9) = 0$ are equal find p and then find the roots of this quadratic equation.

LONG ANSWER TYPE QUESTIONS

20. A train travels at a certain average speed of 54 km and then travels a distance of 63 km at an average speed of 6 km/hr more than the first speed. If it takes 3 hours to complete the total journey, what is its first speed?

21. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

22. A theif runs with a uniform speed of 100 m/minutes. After one minute a policeman runs after the thief to catch him. He goes with a speed of 10 m/minute in the first minute and increases his speed by 10 m/minute every succeeding minute. After how many minutes the policemen will catch the thief?

23. Two water taps together can fill a tank in 6 hours. The tap of larger diameter takes 9 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

24. In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed, so that the area of the grass surrounding the pond would be 1184 m^2. Find the lenght and breadth of the pond.

25. A farmer wishes to grow a 100 m^2 recangular garden. Since he has only 30 m barbed wire, he fences three sides of the rectangular garden letting compound wall of this house act as the fourth side fence. Find the dimensions of his garden.

26. A peacock is sitting on the top of a pillar, which is 9 m high. From a point 27 m away from the bottom fo a pilar, a snake is coming to its hole at the base of the

pillar. Seeing the snake the peacock pounces on it. If their speeds are equa, at what distance from the hole is the snake caught?

27. If the price of a book is reduced by ₹ 5, a person can buy 5 more books for ₹ 300. Find the original list price of the book.

28. ₹ 6500 were divded equally among a certain number of persons. Had there been 15 more persons, each would have got ₹ 30 less. Find the original number of persons.

29. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed was reduced by 200 km/hr and the time of flight increased by 30 minutes. Find the duration of flight.

30. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/hr less than the fast train, find the speeds of the two trains.

31. The speed of a boat in still water is 15 km/hr. It can go 30 km upstream and return downstream to the orignal point in 4 hrs 30 minutes. Find the speed of the stream.

32. Sum of areas of two squares is 400 cm^2. If the difference of their perimeter is 16 cm. Find the side of each square.

33. The area of an isoscles triangle is 60 cm^2. The length of equal sides is 13 cm find length of its base.

34. The denominator of a fraction is one more than twice the numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$. Find the fraction.

35. A girl is twice as old as her sister. Four years hence, the product of their ages (in years) will be 160. Find their present ages.

36. A two digit number is such that the product of its digits is 18. When 63 is subtracted from the number, the digit interchange their places. Find the number.

CBSE 2006

37. Three consecutive positive integers are such that the sum of the square of the first and the product of other two is 46, find the integers. **CBSE 2010**

38. A piece of cloth costs ₹ 200. If the piece was 5 m longer and each metre of cloth costs ₹ 2 less than the cost of the piece would have remained unchanged. How long is the piece and what is the original rate per metre?

39. A motor boat whose speed is 24 km/hr in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream

(CBSE 2016)

40. If the roots of the quadratic equation $(b-c)x^2 + (c-a)x + (a-b) = 0$ are equal, prove $2b = a + c$.

41. If the equation $(1+m^2)n^2x^2 + 2mncx + (c^2 - a^2) = 0$ ha equal roots, prove that $c^2 = a^2(1+m^2)$.

ANSWERS AND HINTS

1. **(*d*)** [$x^4 + 4x^2 + 4x^3 = x^4 + 3 + 4x^2 \Rightarrow 4x^3 = 3 \Rightarrow$ degree = 3]

2. **(*b*)** [Check by substituting $x = 2$ in the equation.]

3. **(*a*)** [Substitute $x = \frac{1}{2}$ in $x^2 + Px - \frac{5}{4} = 0$.]

4. **(*c*)** [$\because$ A quadratic polynomial is of degree 2 and it has atmost two zeroes.]

5. **(*d*)** [$x(x - 7) = 0 \Rightarrow x = 0, x = 7$.]

6. (*a*) [$r = \frac{q^2}{4p}$ ($D = 0 \Rightarrow q^2 - 4pr = 0$)]

(*b*) $p = -6, q = 1$ [$x^2 - 5x - 6 = 0 \Rightarrow (x - 6)(x + 1) = 0$]

(*c*) $K < 4$

(*d*) $c = 4$ ($\because$ product $= 1 \Rightarrow \frac{C}{A} = 1 \Rightarrow \frac{C}{4} = 1$)

(*e*) Linear equation ($x = 0 \Rightarrow ax^2 + bx + c = 0$ reduces to $bx + c = 0$)

7. (*a*) False (A quadratic equation has atmost two real root).

(*b*) True (Coefficient of $x^2 = a$, Constant $= -c$, $D = b^2 - 4ac = b^2 - 4(a)(-c) = b^2 + 4ac > 0$)

(*c*) False ($x^2 = 0.9 \Rightarrow x = \pm\sqrt{0.9}$)

(*d*) False (Degree of quadratic polynomial is 2 not 1 $\because$ Not a straight line)

8. (*i*) $\rightarrow$ *d*

(*ii*) $\rightarrow$ *a*

(*iii*) $\rightarrow$ *e*

(*iv*) $\rightarrow$ *c*

(*v*) $\rightarrow$ *b*

9. $D = 0$ $\quad 20p^2 - 60p = 0, p \neq 0$

$20p(p - 3) = 0$

$p = 3$

10. (*a*) $x = \frac{7}{2}, x = -\frac{3}{4}$ (*b*) $x = \sqrt{5}, x = \frac{-2\sqrt{5}}{3}$

(*c*) $x = \frac{\sqrt{2}}{3}, x = \frac{\sqrt{2}}{3}$ (*d*) $x = \frac{a}{2}, x = -a$

(*e*) $x = -\sqrt{3}, x = \frac{-7\sqrt{3}}{3}$ (*f*) $x = -\sqrt{2}, x = \frac{-5\sqrt{2}}{2}$

(*g*) Take $(x - 1) = y$

$y^2 - 5y - 6 = 0 \Rightarrow (y + 1)(y - 6) = 0$

$y = -1, y = 6$

$x - 1 = -1, x - 1 = 6$

$x = 0, x = 7$

11. $2(-5)^2 + p(-5) - 15 = 0 \Rightarrow p = 7$

$\therefore 7x^2 + 7x + k = 0, \quad D = 49 - 28k = 0$

$\Rightarrow k = \frac{49}{28} = \frac{7}{4}$

12. Sub, $x = \frac{2}{3}$ to get $4a + 9b = -42$ ...(1)

Sub, $x = -3$ to get $9a + b = -21$ ...(2)

Solve (1) and (2) to get $a = 3, b = -6$.

13. Product $= \frac{c}{a} = \frac{4p}{p} = 4,$ sum $= \frac{-b}{a} = \frac{-6}{p}$

ATQ $= \frac{-6}{p} = 4 \quad \Rightarrow \quad P = \frac{-6}{4} = \frac{-3}{2}$

14. $x^2 + (x + 4)^2 = 656$

$x^2 + 4x - 320 = 0$

$D = 1296 \quad x = \frac{-4 \pm \sqrt{1296}}{2} = \frac{-4 + 36}{2}, \frac{-4 - 36}{2}$

$x = \frac{32}{2} = 16$, (rejecting –ve value)

Sides are 16 cm, 20 cm

15. ATQ $\alpha - \beta = 11$

Solve to get $\alpha = 8, \beta = 3$

Sum of roots $\alpha + \beta = \frac{-b}{a} = 5$

Product of roots $= \frac{c}{a}$

$24 = 3k - 3$

$27 = 3k \quad \Rightarrow \quad k = 9$ Ans.

16. $x^2 + kx + 64 = 0 \rightarrow D_1 = k^2 - 256 \geq 0, \quad k^2 \geq 256$

$\Rightarrow \quad k \geq 16 \quad ...(1)$

$k \leq -16$

$x^2 - 8x + k = 0 \rightarrow D_2 = 64 - 4k \geq 0$

$\Rightarrow \quad k \leq 16 \quad ...(2)$

(1) and (2) gives $k = 16$

17. (*a*) $\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\frac{x-a-b-x}{(a+b+x)x} = \frac{a+b}{ab}$

$-(a+b)\,ab = (a+b)\,(a+b+x)\,x$

$x^2 + xa + bx + ab = 0$

$(x+a)\,(x+b) = 0, x = -a, x = -6$

(*b*) $\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\frac{x-a-b-x}{(a+b+x)x} = \frac{a+b}{ab}$

$-(a+b)\,ab = (a+b)\,(a+b+x)\,x$

$x^2 + xa + bx + ab = 0$

$(x+a)\,(x+b) = 0, x = -a, x = -6$

(*c*) Take LCM to get $2x^2 + 5x + 3 = 0, x = -1, x \neq \frac{-3}{2}$.

(*d*) $(4x^2 + 4bx + b^2) - a^2 = 0$

$(2x + b)^2 - a^2 = 0$ apply $A^2 - B^2 = (A + B)(A - B)$

Ans. $x = -\frac{(a+b)}{2}, x = \frac{a-b}{2}$

(*e*) Take LCM to get $3x^2 - 13x + 12 = 0$

Ans. $x = 3, \frac{4}{3}$

(*f*) $4x^2 - 2a^2x - 2b^2x + a^2b^2 = 0$

$2x(2x - a^2) - b^2(2x - a^2) = 0 \Rightarrow (2x - b^2)(2x - a^2) = 0$

$x = \frac{b^2}{2}, \frac{a^2}{2}$

(*g*) Take LCM to get $11x^2 - 21x - 92 = 0$

$11x^2 - 44x + 23x - 92 = 0$. Solve and get

$x = 4, x = \frac{-23}{11}$

(*h*) $\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$

Let $\frac{2x}{x-5} = y \quad \therefore y^2 + 5y - 24 = 0$. Solve to get $y = 3, y = -8$

Sub, $\frac{2x}{x-5} = 3, \frac{2x}{x-5} = -8$

Ans. $x = 15, x = 4$

(*i*) $4x^2 - 4a^2x + a^4 - b^4 = 0$

$(2x - a^2)^2 - (b^2)^2 = 0$

$(2x - a^2 - b^2)(2x - a^2 + b^2) = 0$

$x = \frac{a^2 + b^2}{2}, \quad x = \frac{a^2 - b^2}{2}$

(*j*) Find $D = b^2(6a^2 - 1)^2$

Use $x = \frac{-B \pm \sqrt{D}}{2A}$ to get answer

Ans. $x = \frac{-b}{2a^2}, -3b$

(*k*) Let $\frac{7x+1}{5x-3} = y$

$\therefore\ 3y - \frac{4}{y} = 11 \Rightarrow 3y^2 - 11y - 4 = 0$. Solve to get

$y = -\frac{1}{3}, y = 4$

Sub y and get $x = 0, 1$

(*l*) Take LCM to get $9x^2 + 3x - 12 = 0$

Solve to get $x = 1, x = -\frac{4}{3}$

(*m*) Take LCM to get $2x^2 - 27x + 88 = 0$

$x = 8, \frac{11}{2}$

(*n*) Take LCM to get $x^2 - 4x - 8 = 0$ (Use quadratic formula)

Ans. $x = 2 \pm 2\sqrt{3}$

(*o*) Take LCM to get $2x^2 - 16x + 23 = 0$

Solve using Quadratic formula

Ans. $x = \frac{-8 \pm 3\sqrt{2}}{2}$

(*p*) $x^2 + 7\sqrt{5}x - 2\sqrt{5}x - 70 = 0$

$\left(x + 7\sqrt{5}\right)\left(x - 2\sqrt{5}\right) = 0$

$x = 2\sqrt{5}, -7\sqrt{5}$

(*q*) $\frac{16-x}{x} = \frac{15}{x+1}$

$x^2 - 16 = 0$

$x = \pm 4$

20. Equation $\frac{54}{x} + \frac{63}{x+6} = 3, x \rightarrow$ speed of train at first, $x + 6 \rightarrow$ Increased speed.

Ans. $x = 36, x \neq -3$.

21. Let the natural number be x.

ATQ $x + 12 = \frac{160}{x}$ to get $\quad x^2 + 12x - 160 = 0$

$(x + 20)(x - 8) = 0$

$x = 8, \quad x \neq -20$

22. Let total time to be n minutes.

Policeman will catch the theif in $(n-1)$ minutes.

Total distance covered by thief = $(100\,x)$ metres ...(1)

(as distance covered in 1 min = 100 min)

Distance covered by policemen

$100 + 110 + 120 + ... +$ to $(n - 1)$ tan ...(2)

(1) and (2) $\Rightarrow 100\,n = \frac{(n-1)}{2}\,[2 \times 100 + (n-2)\,10]$

Solve and get $\quad n^2 - 3n - 18 = 0$

$n = 6, \quad n \neq -3$

Policeman will catch the thief in 5 minutes.

23. Time taken by top of smaller diameter = x hrs

Time taken by larger tap = $(x - 9)$ hrs

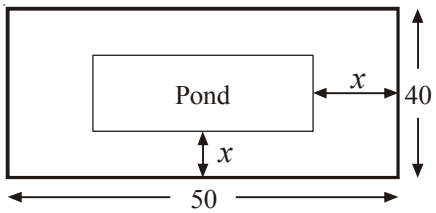
ATQ $\frac{1}{x} + \frac{1}{x-9} = \frac{1}{6}$ and get $x^2 - 21x + 54 = 0$

Ans. $x = 3, x = 18$

$x = 3$ rejeced as $x - 9 = -6 < 0$

$\therefore\ x = 18$ hrs $x - 9 = 18 - 9 = 9$ hrs

24.



Length of rectangular lawn = 50 m

Breadth of rectangular lawn = 40 m

Length of pond = $50 - 2x$

Breadth of pond = $40 - 2x$

Area of lawn – Area of pond = area of grass

$50 \times 40 - (50 - 2x)(40 - 2x) = 1184$

get $x^2 - 45x + 296 = 0$

$x = 37, x = 8$

$x = 37$ rejected $\because$ $40 - 2x = 40 - 2(37) < 0$

Ans. Length of pond = 34 m

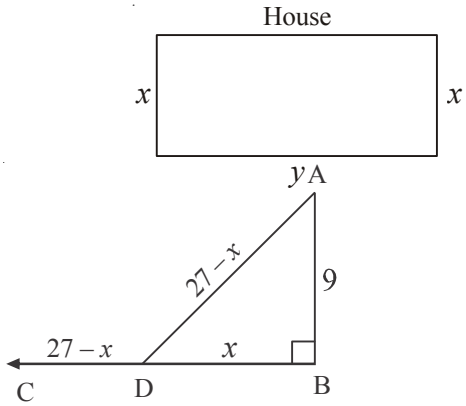
Breadth of pond = 24 m

25. $x + y + x = 30$, $xy = 100$

Solve x = 5m, 10 m,

y = 20 m, 10 m

26.



In ΔABD, pythogorus theorem $9^2 + x^2 = (27 - x)^2$. Solve it to get x = 12 m.

27. Let original list price = ₹ x

ATQ $\frac{300}{x-5} - \frac{300}{x} = 5$

Solve and get $x = 20$, $x = -15 \rightarrow$ rejected

Ans. ₹ 20

28. Let original number of persons be x

ATQ $\frac{6500}{x} - \frac{6500}{x+15} = 30$

Solve and get $x = 50$, $x \neq -65$.

29. ATQ $\frac{600}{x-200} - \frac{600}{x} = \frac{1}{2}$ [Speed of slow train = x km/hr]

Solve to get $x = 600$, $x \neq -400$

Duration of flight $\frac{600}{600}$ = 1hr.

30. ATQ $\frac{600}{x} - \frac{600}{x+10} = 3$ (Speed of slow train x km/hr)

Solve to get $x = 40$, $x \neq -50$

Ans. 5 km/hr

31. ATQ $\frac{30}{15-x}+\frac{30}{15+x}=\frac{9}{2}$ (Speed of stream x km/hr)

Solve to get x = 5, $x \neq -5$

Ans. 5 km/hr

32. $x^2 + y^2 = 400$ ...(1)

$4x - 4y = 16 \Rightarrow x - y = 4$ ...(2)

$y - x = 4$ ...(3)

Solve (1) and (2) to get $x = 16$, $x \neq -12$

Solve (1) and (3) to get $x = 12$, $x \neq -16$

Ans. $x = 16$ m, $y = 12$ m from (1) and (2)

$x = 12$ m, $y = 16$ m from (1) and (3)

33. BC = 2x, BD = x

Use pythagoreas to get

$AD = \sqrt{169 - x^2} = 60$

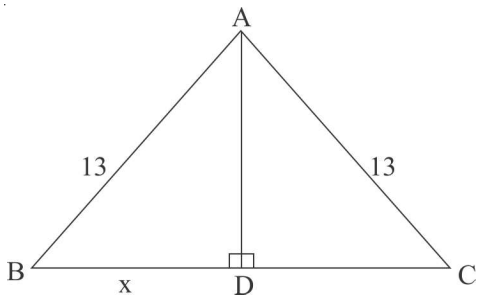
$A = \frac{1}{2} \times 2x \times \sqrt{169 - x^2} = 60$

Solve to get $x^2 = 144$, $x^2 = 25$

$x = 12$ or x = 5

$x \neq -12, -5$

base $2x = 24$, 10 cm



34. Fraction is $\frac{x}{2x+1}$

ATQ $\frac{x}{2x+1}+\frac{2x+1}{x} = 2\frac{16}{21} = \frac{58}{21}$

Solve to get $x = 3$, $x \neq \frac{-7}{11}$

Ans. Fraction = $\frac{3}{7}$.

35. Age of sister = x years

Age of girl = $2x$

ATQ $(x + 4)(2x + 4) = 160$

Solve to get $x^2 + 6x - 72 = 0$

Ans. $x = 6$ years, $x \neq -12$

$2x = 12$ years

36. Let tens place digit = x, then units digits = $\frac{18}{x}$.

No, $10x + \frac{18}{x}$

ATQ $\left(10x + \frac{18}{x}\right) - \left(\frac{10 \times 18}{x} + x\right) = 63$

Solve to get $x = 9, x \neq -2$.

Ans. No. 92

37. Let no. be x, $x + 1$, $x + 2$

ATQ $(x)^2 + (x + 1)(x + 2) = 46$

To get $2x^2 + 3x - 44 = 0$

Use quadratic formula to solve q get $x = 4, x \neq -\frac{22}{4}$

$\therefore$ No.s are 4, 5, 6.

38. Let length of piece be x metre.

ATQ $\frac{200}{x} - \frac{200}{x+5} = 2$

Solve to get $x^2 + 5x - 500 = 0$

Solve to get $x = 20, x \neq -25$

Rate per meter = $\frac{200}{x} = \frac{200}{20} = ₹\ 10$

39. Let speed of boat = x

ATQ $\frac{32}{24-x} - \frac{32}{24+x} = 1$

$x^2 - 64x - 576 = 0$

$(x - 72)(x + 8) = 0$

$x \neq -8$

$x = 72$ km/hr

40. Find D and let $D = 0$

$(c - a)^2 - 4(b - c)(a - b) = 0$

Solve to get $(a + c - 2b)^2 = 0$

$\therefore\ a + c = 2b$

41. $D = 0$

$(2\ mnc)^2 - 4(1 + m^2)\ n^2 (c^2 - a^2) = 0$

to get $4n^2c^2 = 4n^2a^2 (1 + m^2)$

$\therefore\ c^2 = a^2 (1 + m^2)$

Practice Test

Quadratic Equations

Time: 1 Hour **M.M : 20**

SECTION-A

1. The value of k is ..... if $x = 3$ is one root of $x^2 - 2kx - 6 = 0$. **1**

2. If the discriminant of $3x^2 + 2x + \alpha = 0$ is double the discriminant of $x^2 - 4x + 2 = 0$ then value of α is **1**

3. If discriminant of $6x^2 - bx + 2 = 0$ is 1 then value of b is **1**

4. $(x - 1)^3 = x^3 + 1$ is quadratic equation. (T/F) **1**

SECTION-B

5. If roots of $x^2 + kx + 12 = 0$ are in the ratio 1 : 3 find k. **2**

6. Solve for x : $21x^2 - 2x + \frac{1}{21} = 0$ **2**

7. Find k if the quadratic equation has equal roots : $kx(x - 2) + 6 = 0$. **2**

SECTION-C

8. Solve using quadratic formula **3**

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

9. For what value of k, $(4 - k)x^2 + (2k + 4)x + (8k + 1) = 0$ is a perfect square. **3**

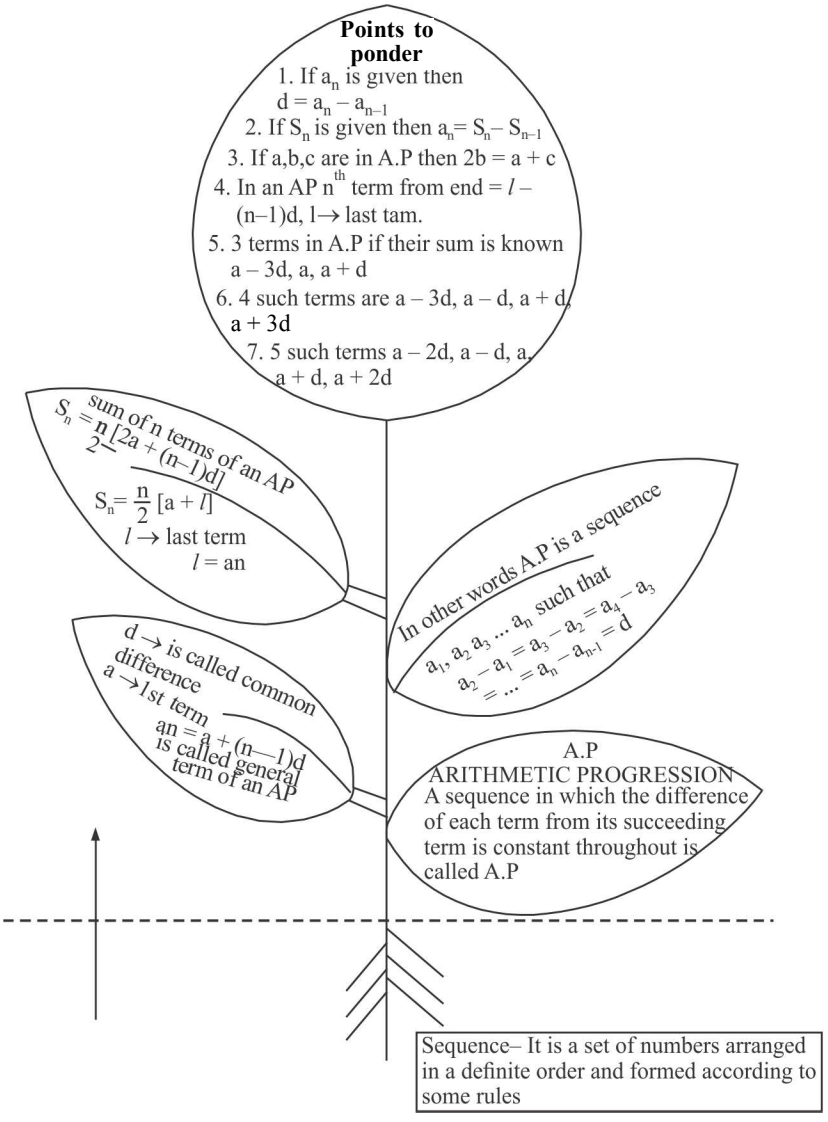
SECTION-C

10. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately. (CBSE 2018) **4**

❐❐❐

CHAPTER

5 Arithmetic Progression



VERY SHORT ANSWER TYPE QUESTIONS

1. FInd 5^{th} term of an A.P. whose n^{th} term is $3n - 5$

2. Find the sum of first 10 even numbers.

3. Write the n^{th} term of odd numbers.

4. Write the sum of first n natural numbers.

5. Write the sum of first n even numbers.

6. Find the n^{th} term of the A.P. $-10, -15, -2, -25, ...$

7. Find the common difference of A.P. $4\frac{1}{9}, 4\frac{2}{9}, 4\frac{1}{3}, ...$

8. Write the common difference of an A.P. whose n^{th} term is $a_n = 3n + 7$

9. What will be the value of $a_8 - a_4$ for the following A.P.
4, 9, 14, ..., 254

10. What is value of a_{16} for the A.P. $-10, -12, -14, -16, ...$

11. $3, k - 2, 5$ are in A.P. find k.

12. For what value of p, the following terms are three consecutive terms of an A.P. $\frac{4}{5}$, p, 2.

13. In the following A.Ps, find the missing terms in the boxes : (NCERT)

(a) 2, ☐, 26 (b) ☐, 13, ☐, 3

(c) 5, ☐, ☐, $9\frac{1}{2}$ (d) -4, ☐, ☐, ☐, ☐, 6

(e) ☐, 38, ☐, ☐, ☐, -22

14. Multiple Choice Questions:

(a) 30th term of the A.P. 10, 7, 4 .... is

(A) 97 (B) 77

(C) –77 (D) –87

(b) 11th term of an A.P. $-3, -\frac{1}{2}, , ...$ is

(A) 28 (B) 22

(C) –38 (D) $-48\frac{1}{2}$

(*c*) In an A.P. if $d = -4$, $n = 7$, $a_n = 4$, then a is

(A) 6 (B) 7

(C) 120 (D) 28

(*d*) The first three terms of an A.P. respectively are $3y - 1$, $3y + 5$ and $5y + 1$ then y equals: **(CBSE 2014)**

(A) –3 (B) 4

(C) 5 (D) 2

(*e*) The list of numbers – 10, – 6, – 2, 2, ... is

(A) An A.P. with $d = -16$ (B) An A.P. with $d = 4$

(C) An A.P. with $d = -4$ (D) Not an A.P.

(*f*) The 11th term from the last term of an A.P. 10, 7, 4, ..., – 62 is **(NCERT)**

(A) 25 (B) –32

(C) 16 (D) 0

(*g*) The famous mathematician associated with finding the sum of the first 100 natural numbers is

(A) Pythagoras (B) Newton

(C) Gauss (D) Euclid

(*h*) What is the common difference of an A.P. in which $a_{18} - a_{14} = 32$?

(A) 8 (B) – 8

(C) – 4 (D) 4

15. Match the following :

Column A	Column B
(*a*) $a = -18$, $n = 10$, $d = 2$ then an of A.P.	(*a*) $\frac{a+c}{2}$
(*b*) a, b and c in A.P. then their Arithmetic mean is	(*b*) 0
(*c*) If 2, 4, 6, are in A.P. then 4, 8, 12 will also be an	(*c*) – 41
(*d*) If $a_n = 9 - 5n$ of an A.P. then a_{10} will be	(*d*) 8
(*e*) If $d = -2$, $n = 5$ and $a_n = 0$ in A.P. then a is	(*e*) A.P.

16. State True/False and justify

(*a*) 301 is a term of A.P. 5, 11, 17, 23 .... (NCERT)

(*b*) Difference of mth and nth term of an A.P. = $(m - n)\,d$.

(*c*) 2, 5, 9, 14, ... is an A.P.

(*d*) Sum of first 20 natural numbers is 410.

(*e*) nth term of A.P. 5, 10, 15, 20 .... n terms and nth term of A.P. 15, 30, 45, 60, ... n terms are same.

SHORT ANSWER TYPE QUESTIONS-I

17. Is 144 a term of the A.P. 3, 7, 11, ... ? Justify your answer.

18. Find the 20^{th} term from the last term of the A.P. 3, 8, 13, ..., 253

19. Which term of the A.P. 5, 15, 25, ... will be 130 more than its 31st term?

20. The first term, common difference and lat term of an A.P. are 12, 6 and 252 respectively, Find the sum of all terms of this A.P.

21. Find the sum of first 15 multiples of 8.

22. Is the sequence formed in the following situations an A.P.

(*i*) Number of students left in the school auditorium from the total strength of 1000 students when they leave the auditorium in batches of 25.

(*ii*) The amount of money in the account every year when Rs. 100 are deposit annually to accumulate at compound interest at 4% per annum.

23. Find the sum of even positive integers between 1 and 200.

24. If $4m + 8, 2m^2 + 3m + 6, 3m^2 + 4m + 4$ are three consecutive terms of an A.P. find m.

25. How many terms of the A.P. 22, 20, 18, ... should be taken so that their sum is zero.

26. If 10 times of 10th term is equal to 20 times of 20th term of an A.P. Find its 30th term.

27. Find the middle term of the A.P. 6, 13, 2, ... 216.

28. Find whether (– 150) is a term of A.P. 11, 8, 5, 2, ... ? **(NCERT)**

29. Find how many two digit numbers are divisible by 6? **(CBSE 2011)**

30. If $\frac{1}{x+2}, \frac{1}{x+3}$ and $\frac{1}{x+5}$ are in A.P. find x. **(CBSE 2011)**

31. Find the middle term of an A.P. – 6, – 2, 2, ... 58. **(CBSE 2011)**

32. In an A.P. find S_n, where $a_n = 5n - 1$. Hence find the sum of the first 20 terms. **(CBSE 2011)**

33. Which term of A.P. 3, 7, 11, 15 .... is 79? Also find the sum 3 + 7 + 11 + ... + 79. **(CBSE 2011C)**

34. Which term of the A.P. : 121, 117, 113 ... is the first negative terms ? **(NCERT)**

35. Find the 20th term from the last term of the A.P. 3, 8, 13, ... 253. **(NCERT)**

SHORT ANSWER TYPE QUESTIONS-II

36. Find the middle terms of the A.P. 7, 13, 19, ..., 241.

37. Find the sum of integers between 10 and 500 which are divisible by 7.

38. The sum of 5th and 9th terms of an A.P. is 72 and the sum of 7^{th} and 12^{th} term is 97. Find the A.P.

39. If the mth term of an A.P. be $\frac{1}{n}$ and n^{th} term be $\frac{1}{m}$, show that its $(mn)^{th}$ is 1.

40. If the p^{th} of term A.P. is q and the q^{th} term is p, prove that its n^{th} term is $(p + q - n)$.

41. Find the number of natural numbers between 101 and 999 which are divisible by both 2 and 5.

42. The sum of 5^{th} and 9^{th} terms of an A.P. is 30. If its 25^{th} term is three times its 8^{th} term, find the A.P.

43. If S_n, the sum of first n terms of an A.P. is given by $Sn = 5n^2 + 3n$, then find its n^{th} term and common difference.

44. Which term of the A.P. 3, 15, 27, 39 .... wil be 120 more than its 21st term?

(CBSE 2018)

45. If Sn, the sum of first n terms of an A.P. is given by $Sn = 3x^2 - 4x$, find the nth term. **(CBSE 2018)**

46. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed? **(NCERT)**

47. For what value of n, are the n^{th} term of two A.P's 63, 65, 67 ..... and 3, 10, 17 ..... are equal ? **(NCERT)**

48. Which term of an A.P. 3, 15, 27, 39 .... will be 132 more than its 54th term?

(NCERT)

49. If the sum of the first 14 terms of an A.P. is 1050 and its first term is 10, find the 20th term. **(NCERT)**

50. Find the sum of odd numbers between 0 and 50. **(NCERT)**

51. If $Sn = 4n - n^2$ in an A.P. find the A.P. **(NCERT)**

52. How many terms of the A.P. 9, 17, 25, ... must be taken to give a sum of 636?

(NCERT)

LONG ANSWER TYPE QUESTIONS

53. The sum of third and seventh terms of an A.P. is 6 and their product is 8. Find the sum of first 16^{th} terms of the A.P.

54. Determine the A.P. whase 4^{th} term is 18 and the difference of 9^{th} term from the 15^{th} term is 30.

55. The sum of first 9 terms of an A.P. is 162. The ratio of its 6^{th} term to its 13^{th} term is 1:2. Find the first and fifteenth terms of the A.P.

56. If the 10th term of an A.P. is 21 and the sum of its first 10 terms is 120, find its n^{th} term.

57. The sum of first 7 terms of an A.P. is 63 and the sum of its next 7 term is 161. Find the 28^{th} term of this A.P.

58. The sum of first 20 terms of an A.P. is one third of the sum of next 20 term. If first term is 1, find the sum of first 30 terms of this A.P.

59. If the sum of the first four terms of an AP is 40 and the sum of the first fourteen terms of an AP is 280. Find the sum of first n terms of the A.P. **(CBSE 2018)**

60. Ramkali required Rs. 2500 after 12 weeks to send her daughter to school. She saved ₹ 100 in the first week and increased her weekly savings by ₹ 20 every week. Find wheather she will be able to send her daughter to school after 12 weeks. **(CBSE 2015)**

61. In an AP of 50 terms, the sum of first 10 terms is 210 and the sum of last 15 terms is 2565. Find the A.P. **(CBSE 2014)**

62. The sum of first n terms of an A.P. is $5n^2 + 3n$. If the mth term is 168, find the value of m. Also find the 20th term of the A.P. **(CBSE 2013)**

63. If the sum of the first seven terms of an A.P. is 49 and the sum of its first 17 terms is 289. Find the sum of first n terms of an A.P. **(CBSE 2016)**

64. If the 4th term of an A.P. is zero, prove that the 25th term of the A.P. is three times its 11th term. **(CBSE 2016)**

65. In an A.P. if $S_5 + S_7 = 167$ and $S_{10} = 235$. Find the A.P., where S_n denotes the sum of its first n terms. **(CBSE 2015)**

66. In an AP prove $S_{12} = 3\ (S_8 - S_4)$ where Sn represent the sum of first n terms of an A.P. **(CBSE 2015)**

VERY SHORT ANSWER TYPE QUESTIONS-I

1. $a_n = 3x - 5 \qquad a_5 = 10$

2. $S_n = \frac{10}{2}\ [2 \times 2 + 9 \times 2] = 110$

3. 1, 3, 5, ...

$a_n = 1 + (n - 1)2 = 2n - 1.$

4. $1 + 2 + ... + n = \frac{n}{2}\ [1 + n]$

5. $2 + 4 + 6 + ... + 2n = \frac{n}{2}[2 + 2n] = n(n + 1)$

6. $a_n = a + (n - 1)d = -5(n + 1)$

7. $d = a_2 - a_1 = \frac{1}{9}$

8. $a_1 = 3 + 7 = 10,\ a_2 = 6 + 7 = 13,\ d = 3$

9. $(a + 7d) - (a + 3d) = 4d = 20$

10. $a_{16} = a + 15d = -40$

11. 3, $k - 2$, 5 are in A.P.

$\therefore\ K - 2 = \frac{3+5}{2} = 4 \qquad K = 6$

12. $P = \frac{7}{5}$ (same as Q.11)

13. (*a*) $\boxed{14}$ (*b*) $\boxed{18}, \boxed{8}$

(*c*) $\boxed{6\frac{1}{2}}, \boxed{8}$ (*d*) $\boxed{-2}, \boxed{0}, \boxed{2}, \boxed{4}$

(*e*) $\boxed{53}, \boxed{23}, \boxed{8}, \boxed{-7}$

14. (*a*) C (*b*) B

(*c*) D (*d*) C

(*e*) B (*f*) B

(*g*) C (*h*) A

15. $(a) \rightarrow (b)$ $\quad (b) \rightarrow (a)$
$(c) \rightarrow (e)$ $\quad (d) \rightarrow (c)$
$(e) \rightarrow (d)$

16. (a) False, $301 = 5 + (n - 1)\,6$

Solving we get $n = \dfrac{151}{3}$ which is not a natural number.

$\therefore$ 301 is not a term of this A.P.

(b) True $[a + (m - 1)\,d] - [a + (n - 1)\,d] = (m - n)\,d$

(c) False $\because\ a_2 - a_1 = 5 - 2 = 3$

$\because\ a_3 - a_2 = 9 - 5 = 4$

(d) False $\because\ S_n = \dfrac{n(n+1)}{2} = \dfrac{20 \times 21}{2} = 210$

(e) True (If a, b, c, d ... are in AP then ka, kb, kc, kd ..... are in AP)

$k \neq 0$

(f) $144 = 3 + (n - 1)\,4$

$\dfrac{141}{4} + 1 = n$ which is not possible

18. No, use $l - (n - 1)\,d$

Ans. 158

19. Let $a_n = 130 + a_{31}$

Solve to get $n = 44$

Ans. 44th term

20. $a = 12, d = 6, a_n = 252 \quad \Rightarrow \quad n = 41$

Find $S_{41} = 5412$, use $S_n = \dfrac{n}{2}\,[2a + (n - 1)\,d]$

21. $S_{15} = \dfrac{15}{2}[2a + 14d]$

where $a = 8, d = 8$

Ans. 960

22. (i) Yes $\quad (ii)$ No

23. $2 + 4 + 6 + ... + 198$

$a = 2, d = 2, a_n = 198 \Rightarrow n = 99$

$S_n = \dfrac{n}{2}[a + l] = 9900$

24. $b = \dfrac{a+c}{2}$

$\therefore\ 2m^2 + 3m + 6 = \dfrac{4m+8+3m^2+4m+4}{2}$

Solve to get $m^2 - 2m = 0$

$m = 0, 2$

25. $S_n = 0 \Rightarrow \dfrac{n}{2}\ [44 + (n-1)\ (-2)] = 0.$

Solve $n = 23$

26. ATQ $10\ a_{10} = 20\ a_{20}$

$\Rightarrow a_{10} = 2a_{20}$

$a + 9d = 2a + 38d$

$a = -29d$ ...(1)

$a_{30} = a + 29d$

Substitute a from (1)

Ans. $a_{30} = 0$

27. 6, 13, 20, ..., 216

Find n from $a_n = a + (n-1)\ d$

then use concept of median

Middle term = 111.

28. Let $a_n = -150$

$11 + (n-1)\ (-3) = -150$

Solve and get n is not a natural number.

$\therefore$ **Ans.** No.

29. Two digit No.s divisible by 6 are 12, 18, 24, ... 96.

$a_2 - a_1 = a_3 - a_2 = 6$

$\therefore$ A.P., $a_n = 96 \Rightarrow n = 15$

30. $\dfrac{2}{x+3} = \dfrac{1}{x+2} + \dfrac{1}{x+5}\ \ (2b = a + c)$

Solve to get $x = 1$.

31. $a_n = a + (n-1)\,d$

$58 = -6 + (n-1)\,4$

find $n = 17$

Find Middle term using conceptof median

$= \left(\frac{n+1}{2}\right)^{\text{th}}$ term = 9th term

$a_9 = -6 + 8(4) = 26$

32. $a_n = 5n - 1$

Find AP $a_1 = 4$, $a_2 = 9$, $a_3 = 14$

4, 9, 14, ...

$a_2 - a_1 = 5 = a_3 - a_2$

$S_n = \frac{n}{2}[2a + (n-1)d] = \frac{n}{2}[8 + (n-1)\,5]$

$= \frac{n}{2}[5n + 3]$

$S_{20} = \frac{20}{2}\,[100 + 3] = 10 \times 103 = 1030$

33. $79 = 3 + (n-1)\,4$

$n = 26$

$S_{26} = \frac{26}{2}\,[3 + 79] = 13[82]$

$S_{26} = 1066$

34. Let $a_n < 0$

$121 + (n-1)\,(-4) < 0$

$121 - 4n + 4 < 0$

$125 < 4n$

$n > \frac{125}{4}$

$\therefore\ n = 32$

32nd term will be first negative term.

35. 20th term from end using $[l - (n-1)\,d]$

$= 253 - 19 \times 5$

$= 253 - 95 = 158$

SHORT ANSWER TYPE QUESTIONS-II

36. Same as Q.27.

Ans. 121, 127

37. No.s between 10 and 500 which are divisible by 7, 14, 21, 28 ..., 497

Find n, using $a_n = a + (n-1)\,d$, then use $S_n = \frac{n}{2}\,[2a + (n-1)\,d]$

Ans. $S_n = 17885$.

38. $a_5 + a_9 = 72$

$a_7 + a_{12} = 97$

Solve these equations to get a and d.

A.P., 6, 11, 16, 21, 26, ...

39. $a_m = \frac{1}{n} \quad \Rightarrow a + (m-1)d = \frac{1}{n}$

$a_n = \frac{1}{m} \quad \Rightarrow a + (n-1)d = \frac{1}{m}$

$\quad\quad - \quad - \quad\quad\quad -$

$$(m-n)\,d = \frac{1}{n} - \frac{1}{m} = \frac{m-n}{mn}$$

$\therefore\ d = \frac{1}{mn}$, find $a = \frac{1}{mn}$

$a_{mn} = a + (mn-1)\,d$

$= \frac{1}{mn} + (mn-1)\frac{1}{mn}$

$a_{mn} = 1$.

40. $a_p = q, \quad a_q = p$

Solve to get a and d then find $a_{p+q-n} = 0$

41. No.s divisible by both 2 and 5

$\Rightarrow$ No.s divisible by 10.

No.s between 101 and 999 divisible by 2 and 5 both 110, 120, 130, 140, ..., 990.

Use $a_n = 990$ to get $n = 89$.

42. ATQ $a_5 + a_9 = 30$

$a_{25} = 3\ a_8$

Solve to get $a = 3$, $d = 2$

A.P. 3, 5, 7, 9, ...

43. $S_n = 5n^2 + 3n$

Find $a_n = S_n - S_{n-1} = 10\ n - 2$

Use it to get $d = 10$

44. Let $a_n = 120 + a_{21}$

$3 + (n - 1)d = 120 + [3 + 20d]$

$3 + (n - 1)12 = 120 + [3 + 20 \times 12]$

$= 120 + 243$

$(n - 1)12 = 363 - 3 = 360$

$n = 31$

45. $S_n = 3n^2 - 4n$

$a_n = S_n - S_{n-1}$

$= (3n^2 - 4n) - [3(n - 1)^2 - 4(n - 1)]$

$= (3n^2 - 4n) - [3n^2 + 3 - 6n - 4n + 4]$

$= - [7 - 6n]$

$a_n = 6n - 7$

46. 23, 21, 19, ... 5

$a_n = a + (n - 1)\ d$

$S = 23 + (n - 1)\ (- 2)$

$n = 10$

47. 63, 65, 67, ...

$a_n = 63 + (n - 1)\ 2$

$= 61 + 2n$

3, 10, 17, ...

$a_n = 3 + (n - 1)\ 7$

$= 7n - 4$

$61 + 2n = 7n - 4$

$65 = 5n$

$n = 13$

48. 65th term **(NCERT)**

49. $S_{14} = 1050,\ a = 10$

$$S_{14} = \frac{14}{2}\ [2 \times 10 + 13d]$$

$$\frac{1050}{7} = 20 + 13d$$

$$\frac{150 - 20}{13} = d \Rightarrow d = 10$$

$$a_{20} = a + 19d = 10 + 190 = 200$$

50. Odd no.s between 0 to 50

1, 3, 5, 7, ..., 49

$a_n = 49$

$a + (n - 1)d = 49$

$1 + (n - 1)2 = 49$

$n = 25$

$$S_n = \frac{n}{2}\ [a + l]$$

$$S_{25} = \frac{25}{2}\ [1 + 49] = 25 \times 25 = 625$$

51. $S_n = 4n - n^2$

$S_1 = a_1 = 4 - 1 = 3$

$S_2 = a_1 + a_2 \Rightarrow a_2 = 1$ AP 3, 1, – 1, ...

$S_3 = a_1 + a_2 + a_3 = -1$

52. $n = 12,\ n = -\frac{53}{4}$ **(NCERT)**

LONG ANSWER TYPE QUESTIONS

53. $a_3 + a_7 = 6$ $\quad a = 1,\ d = \frac{1}{2},\ S_n = 76$

will give

$a_3 \times a_7 = 8$ $\quad a = 5,\ d = -\frac{1}{2},\ S_n = 20$

Ans. 76, 20

54. **ATQ** $a_4 = 18$ ...(1), $a_{15} - a_9 = 30$ ...(2)

equation (2) will give $d = 5$

Substitute $d = 5$ in (1) to get $a = 3$

A.P. 3, 8, 13, ...

55. ATQ $S_9 = 162 \Rightarrow \frac{9}{2}\,[2a + 8d] = 162$ ...(1)

ATQ $\frac{a_6}{a_{13}} = \frac{1}{2}$ solve and get $a = 2d$

Sub $a = 2d$ in (1) to get $d = 3$, $a = 6$

$a_{15} = a + 14d$

Ans. $a_{15} = 48$

56. $a_{10} = 21$, $S_{10} = 120$. Solve these to get a and d then find

$a_n = a + (n - 1)d$

Ans. $a_n = 2n + 1$

57. ATQ $S_7 = 63$,

...(1)

Sum of next 7 terms = $S_{14} - S_7 = 161$ ...(2)

Use $S_n = \frac{n}{2}\,[2a + (n - 1)\,d]$

Solve (1) and (2) to get a and d then find a_{28} using $a_n = a + (n - 1)\,d$.

Ans. $a_{28} = 57$

58. ATQ $S_{20} = \frac{1}{3}(S_{40} - S_{20})$, $a = 1$

Use $S_n = \frac{n}{2}\,[2a + (n - 1)\,d]$ and $a = 1$ to find d

then find S_{30}.

Ans. 900

59. $S_4 = 40 \Rightarrow \frac{4}{2}\,[2a + 3d] = 40$

$S_{14} = 280 \Rightarrow \frac{14}{2}\,[2a + 13d] = 280$

Solve to get $a = 7$, $d = 2$

60. $a = 100, d = 20, n = 12$

$S_{12} = \frac{12}{2}\ [200 + 220] = 6 \times 420$

$= 2520 > 2500$

$\therefore$ Ram kali will be able to send her daughter to school after 12 weeks.

61. $S_{10} = 210 \Rightarrow 5\ [2a + 9d] = 210$

$2a + 9d = 42$ ...(1)

$S_{50} - S_{35}\ 2565 \Rightarrow \frac{50}{2}[2a + 49d] - \frac{35}{2}[2a + 34d] = 2565$

$\frac{15}{2}\ (2a) + d\ [25 \times 49 - 35 \times 17] = 2565$

$15a + d\ [1225 - 595] = 2565$

or $15a + 630\ d = 2565$

or $3a + 126d = 513$ ...(2)

Solve (1) and (2) $d = 4, a = 3$.

62. $S_n = 5n^2 + 3n$

$S_1 = a_1 = 8$

$S_2 = a_1 + a_2$

$26 = 8 + a_2 \Rightarrow a_2 = 18$

$d = 18 - 8 = 10$

$a_m = 168 \quad \Rightarrow \quad a + (m - 1)d = 168$

$8 + (m - 1)10 = 168 \Rightarrow m = 17$

$a_{20} = a + 19d = 8 + 190 = 198$

63. $S_7 = 49, S_{17} = 289$ (Solve just like Q 53.)

64. $a_4 = 0 \quad \Rightarrow \quad a + 3d = 0 \quad \Rightarrow \quad a = -3d$

$a_{25} = a + 24d = -3d + 24d = 21d$

$a_{11} = a + 10d = -3d + 10d = 7d \qquad a_{25} = 3a_{11}$

65. Use $S_n = \frac{n}{2}\ [2a + (n - 1)\ d]$

Solve like in Q.53.

66. L.H.S. $= S_{12} = \frac{12}{2}\ [2a + 11d] = 6\ [2a + 11d]$

R.H.S. $= 3\left[\frac{8}{2}(2a + 7d) - \frac{4}{2}(2a + 3d)\right] = 3[4a + 22d] = 6[2a + 11d]$

Practice Test

Arithmetic Progression

Time: 1 Hr. **M.M. : 20**

Section-A

1. Find the sum of first 10 natural numbers. **1**

2. What is the common difference of an A.P. $8\frac{1}{8}, 8\frac{2}{8}, 8\frac{3}{8}, ...$ **1**

3. If k, $2k - 1$ and $2k + 1$ are in A.P. them value of k is **1**

4. The 10th term from the end of the AP 8, 10, 12, ..., 126 is **1**

Section-B

5. How many 2 digit number are there in between 6 and 102 which are divisible by 6. **2**

6. The sum of n terms of an A.P. is $n^2 + 3n$. Find its 20^{th} term. **2**

7. Find the sum $(-5) + (-8) + (-11) + ... + (-230)$ **2**

Section-C

8. Find the five terms of an A.P. whose sum is $12\frac{1}{2}$ and first and last term ratio is 2 : 3. **3**

9. Find the middle term of an A.P. 20,16,12,.....,– 176. **3**

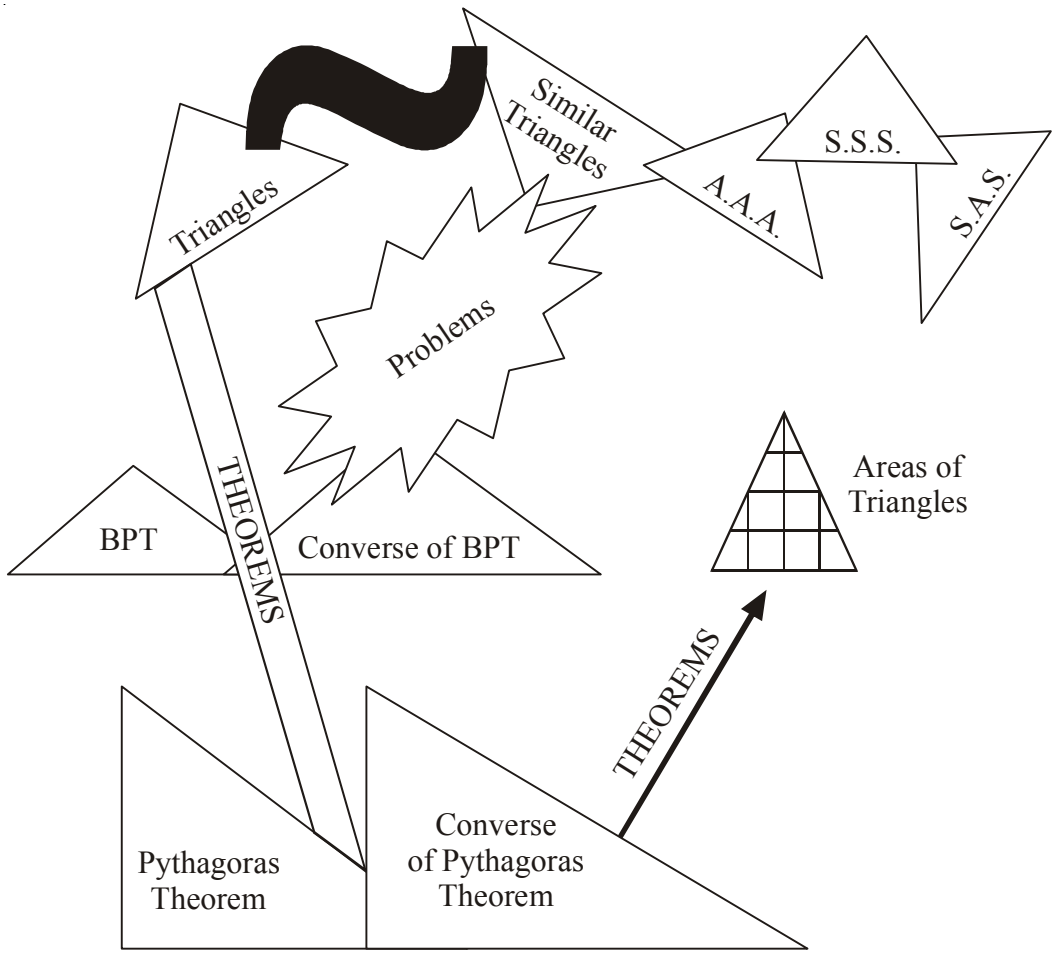
Section-D

10. The sum of three numbers in A.P. is 24 and their product is 440. Find the numbers. **4**

❐❐❐

CHAPTER

6 Triangles



Key Points:

1. **Similar Triangles:** Two triangles are said to be similar if their corresponding angles are equal and their corresponding sides ar proportional.
2. **Criteria for Similarity:**
   in ΔABC and ΔDEF
   (*i*) **AAA Similarity :** $\Delta ABC \sim \Delta DEF$ when $\angle A, \angle D, \angle B = \angle E$ and $\angle C = \angle F$
   (*ii*) SAS Similarity :
   $\Delta ABC \sim \Delta DEF$ when $\frac{AB}{DE} = \frac{BC}{EF}$ and $\angle B = \angle E$
   (*iii*) SSS Similarity : $\Delta ABC \sim \Delta DEF$, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$
3. **The proof of the following theorems can be asked in the examination :**
   (*i*) Basic Proportionality Theorem : If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio.
   (*ii*) The rato of aras of two similar triangles is equal to the square of the ratio of their corresponding sides.
   (*iii*) Pythagoras Theorem: In a right triangles the square of the hypotenuse is equal to the sum of the squares of the other two sides.
   (*iv*) Converse of pythagoras theorem : In a triangle, if the square of one side is equal to the sum of squares of other sides then the angle opposite to the first side is a right angle.

VERY SHORT ANSWER TYPE QUESTIONS

1. **Fill in the blanks :**
   (*i*) All equilateral triangles are _____ .
   (*ii*) If $\Delta ABC \sim \Delta FED$, then $\frac{AB}{\quad} = \frac{\quad}{ED}$.
   (*iii*) Circles with equal radii are _____ .
   (*iv*) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the _____ ratio.
   (*v*) In _____ triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

2. State True or False :

(*i*) All the similar figures are always congruent.

(*ii*) The Basic Proportionality Theorem was given by Pythagoras.

(*iii*) The mid-point theorem can be proved by Basic Proprotionality Theorem.

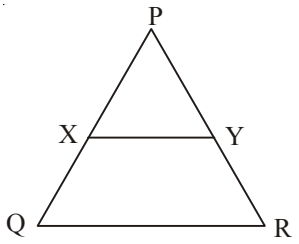
(*iv*) Pythagoras Theorem is valid for right angled triangle.

(*v*) If the sides of two similar triangles are in the ratio 4 : 9, then the areas of these triangles are in the ratio 16 : 81.

3. Match the following :

Column I	Column II
(*a*) If corresponding angles are equal in two triangles, then the two triangles are similar.	(*i*) SAS similarity criterion
(*b*) If sides of one triangle are proportional to the sides of the other triangle, then the two triangles are similar.	(*ii*) ASA similarity criterion
(*c*) If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.	(*iii*) AAA similarity criterion
	(*iv*) SSS similarity criterion

4. In the following figure, XY || QR and $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$, then



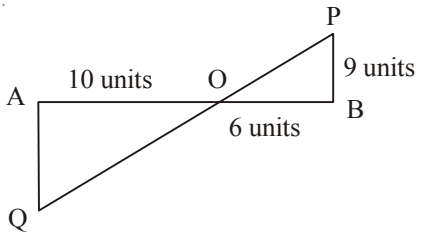
(*a*) XY = QR

(*b*) $XY = \frac{1}{3}QR$

(*c*) $XY^2 = QR^2$

(*d*) $XY = \frac{1}{2}QR$

5. In the following figure, QA ⊥ AB and PB ⊥ AB, then AQ is



(*a*) 15 units (*b*) 8 units

(*c*) 5 units (*d*) 9 units

6. The ratio of areas of two similar triangles is equal to the

(*a*) ratio of their corresponding sides.

(*b*) ratio of their corresponding altitudes.

(*c*) ratio of the square of their corresponding sides.

(*d*) ratio of their perimeter.

7. The areas of two similar triangles are 144 cm^2 and 81 cm^2. If one median of the first triangleis 16 cm, length of corresponding median of the second triangle is

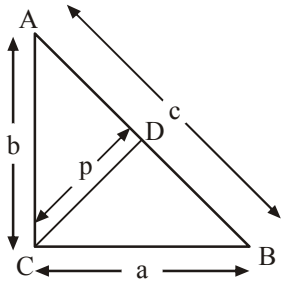
(*a*) 9 cm (*b*) 27 cm

(*c*) 12 cm (*d*) 16 cm

8. In a right triangle ABC, in which $\angle C = 90°$ amd CD $\perp$ AB. If BC = *a*, CA = *b*, AB = *c* and CD = *p*, then

(*a*) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$ (*b*) $\frac{1}{p^2} \neq \frac{1}{a^2} + \frac{1}{b^2}$

(*c*) $\frac{1}{p^2} < \frac{1}{a^2} + \frac{1}{b^2}$ (*d*) $\frac{1}{p^2} > \frac{1}{a^2} + \frac{1}{b^2}$



9. If $\Delta ABC \sim \Delta DEF$, ar($\Delta DEF$) = 100 cm^2 and $\frac{AB}{DE} = \frac{1}{2}$, then ar(DABC) is

(*a*) 50 cm^2 (*b*) 25 cm^2

(*c*) 4 cm^2 (*d*) 200 cm^2

10. If the three sides of a triangle are *a*, $\sqrt{3}a$ and $\sqrt{2}a$, then the measure of hte angle opposite to longest side is

(*a*) 45° (*b*) 30°

(*c*) 60° (*d*) 90°

11. A vertical pole of length 3 m casts a shadow of 7 m and a tower casts a shadow of 28 m at a time. The height of tower is

(*a*) 10 m (*b*) 12 m

(*c*) 14 m (*d*) 16 m

12. The lengths of the diagonals of a rhombus are 16 cm and 12 cm. Then, the length of the side of the rhombus is **(NCERT Exempler)**

(*a*) 9 cm (*b*) 10 cm

(*c*) 8 cm (*d*) 20 cm

13. If $\Delta ABC \sim \Delta EDF$ and ΔABC is not similar to ΔDEF, then which of the following is not true? **(NCERT Exempler)**

(*a*) BC.EF = AC.FD (*b*) AB.EF = AC.DE

(*c*) BC.DE = AB.EF (*d*) BC.DE = AB.FD

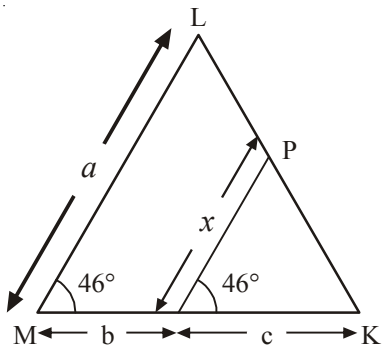
14. Write the statement of pythagoras theorem.

15. Write the statement of Basic Proportionality Theorem.

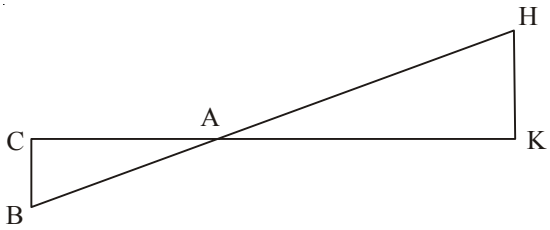
16. Is the triangle with sides 12 cm, 16 cm and 18 cm a right triangle?

17. If $\Delta ABC \sim \Delta QRP$, $\frac{\text{Area } (\Delta ABC)}{\text{Area } (\Delta PQR)} = \frac{9}{4}$, AB = 18 cm, BC = 15 cm, then find the length of PR. **(CBSE 2018)**

18. In the given Fig., $\angle M = \angle N = 46°$, Express *x* in terms of *a*, *b* and *c*.



19. In the given Fig. $\Delta AHK \sim \Delta ABC$. If AK = 10 cm, BC = 3.5 cm and HK = 7 cm, find AC. **(CBSE 2010)**



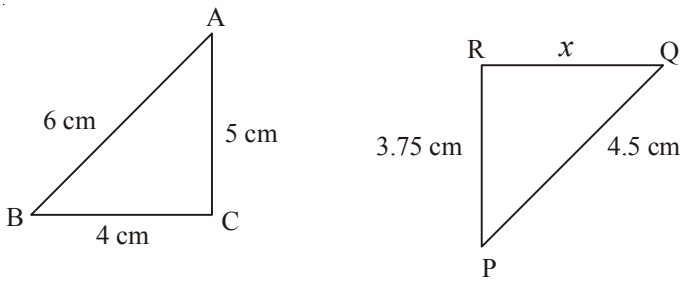
20. It is given that $\Delta DEF \sim \Delta RPQ$. Is it true to say that $\angle D = \angle R$ and $\angle F = \angle P$?

21. If the corresponding Medians of two similar triangles are in the ratio 5 : 7. Then find the ratio of their sides.

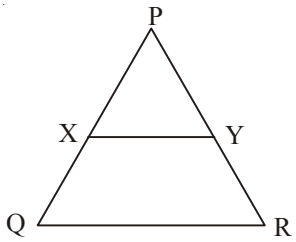
22. An aeroplane leaves an airport and flies due west at a speed of 2100 km/hr. At the same time, another aeroplane leaves the same place at airport and flies due south at a speed of 2000 km/hr. How far apart will be the two planes after 1 hour?

23. The areas of two similar ΔABC and ΔDEF are 225 cm^2 and 81 cm^2 respectively. If the longest side of the larger triangle ΔABC be 30 cm, find the longest side of the smaller triangle DEF.

24. In the given figure, if $\Delta ABC \sim \Delta PQR$, find the value of x?

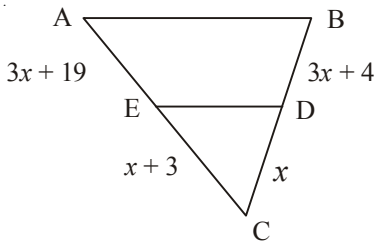


25. In the given figure, XY || QR and $\frac{PX}{XQ} = \frac{PY}{YR} = \frac{1}{2}$, find XY : QR.



26. In the given figure, find the value of x which will make DE || AB ?

(NCERT Exempler)



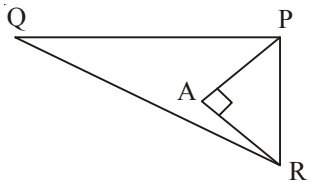
27. If $\Delta ABC \sim \Delta DEF$, BC = 3EF and ar (DABC) = 117cm^2 find area (ΔDEF).

28. If ΔABC and ΔDEF are similar triangles such that $\angle A = 45°$ and $\angle F = 56°$, then find the ratio of their corresponding attitudes.

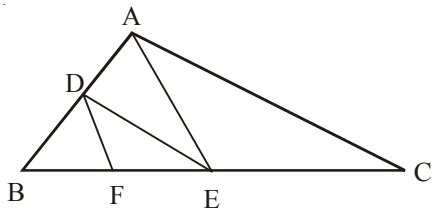
29. If the ratio of the corresponding sides of two similar triangles is 2 : 3, then find the ratio of their corresponding attitudes.

SHORT ANSWER TYPE QUESTIONS-I

30. In the given Fig. PQ = 24 cm, QR = 26 cm, $\angle PAR = 90°$, PA = 6 cm and AR = 8 cm, find $\angle QPR$.

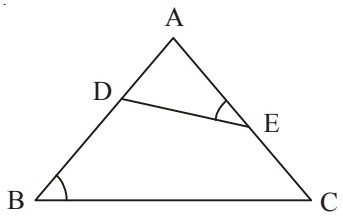


31. In the given Fig., DE || AC and DF || AE. Prove that $\frac{FE}{BF} = \frac{EC}{BE}$

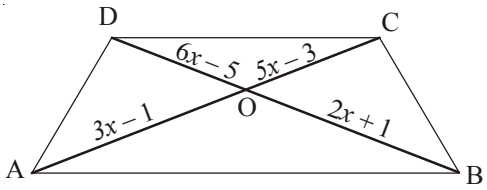


32. In ΔABC, AD $\perp$ BC. Such that $AD^2 = BD \times CD$. Prove that ΔABC is right angled triangle.

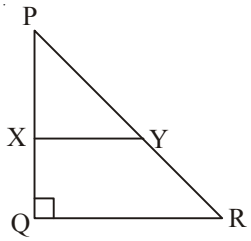
33. In the given Fig., D and E are points on sides AB and CA of ΔABC such that $\Delta B = \angle AED$. Show that $\Delta ABC \sim \Delta AED$.



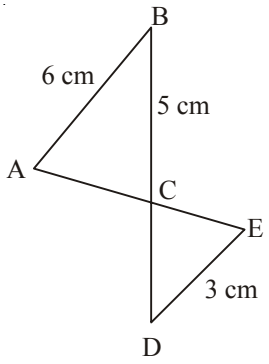
34. In the given fig., AB || DC and diagonals AC and BD intersects at O. If OA = $3x - 1$ and OB = $2x + 1$, OC = $5x - 3$ and OD = $6x - 5$, find the value of x.



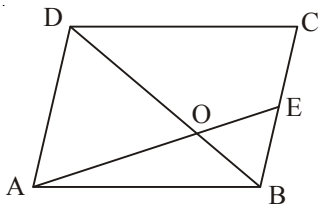
35. In the given Fig. PQR is a triangle, right angled at Q. If XY || QR, PQ = 6 cm, PY = 4 cm and PX : XQ = 1 : 2. Calculate the lengths of PR and QR.



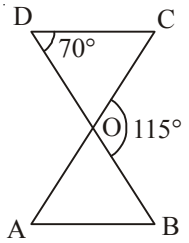
36. In the given figure, AB || DE. Find the length of CD.



37. In the given figure, ABCD is a parallelogram. AE divides the line segment BD in the ratio 1 : 2. If BE = 1.5 cm find BC.



38. In the given figure, $\Delta ODC \sim \Delta OBA$, $\angle BOC = 115°$ and $\angle CDO = 70$. Find, (*i*) $\angle DOC$, (*ii*) $\angle DCO$, (*iii*) $\angle OAB$, (*iv*) $\angle OBA$.

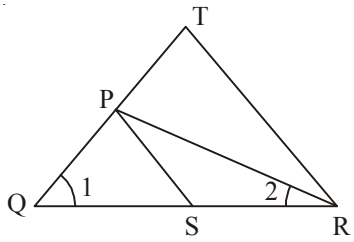


39. Perimeter of two equilateral triangles ABC and PQR are 144 m and 96 m, Find ar (ΔABC) : ar (ΔPQR).

SHORT ANSWER TYPE QUESTIONS-II

40. In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$ then prove that $\Delta PQS \sim \Delta TQR$.

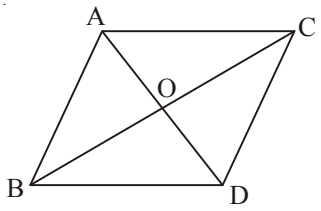
(NCERT)



41. In equilateral ΔABC, $AD \perp BC$. Prove that $3BC^2 = 4AD^2$.

42. In ΔABC, $\angle ACB = 90°$ and $CD \perp AB$. Prove that $\frac{BC^2}{AC^2} = \frac{BD}{AD}$. (HOTS)

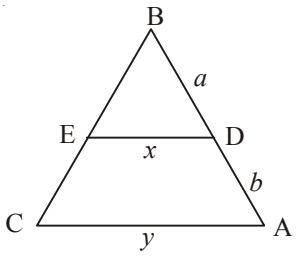
43. In the adjoining figure ΔABC and ΔDBC are on the same base BC. AD and BC intersect at O. Prove that $\frac{\text{area}\,(\Delta ABC)}{\text{area}\,(\Delta DBC)} = \frac{AO}{DO}$.



44. If AD and PS are medians of ΔABC and ΔPQR respectively where $\Delta ABC \sim \Delta PQR$, Prove that $\frac{AB}{PQ} = \frac{AD}{PS}$.

45. In the given figure, DE || AC. Which of the following is correct?

$x = \frac{a+b}{ay}$ or $x = \frac{ay}{a+b}$

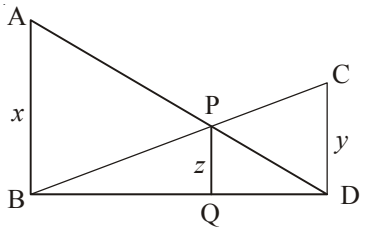


46. Prove that the sum of the square of the sides of a rhombus is equal to the sum of the squares of its diagonals. **(NCERT, CBSE 2019)**

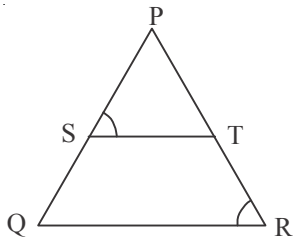
47. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3 m, find how for she is away from the base of the pole. **(NCERT Exempler)**

48. Two poles of height a metrs and b metres are p metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is gives by $\frac{ab}{a+b}$ metres.

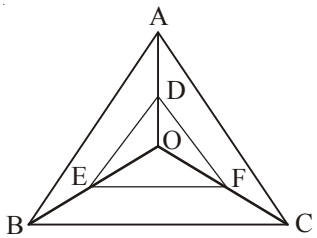
49. In the given figure AB || PQ || CD, AB = x, CD = y and PQ = z. Prove that $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$.



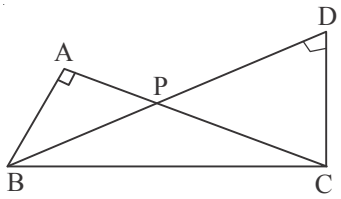
50. In the given figure $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$. Prove that PQR is an isoscles triangle. **(NCERT)**



51. In the figure, a point O inside ΔABC is joined to its vertices. From a point D on AO, DE is drawn parallel to AB and from a point E on BO, EF is drawn parallel to BC. Prove that DF || AC.



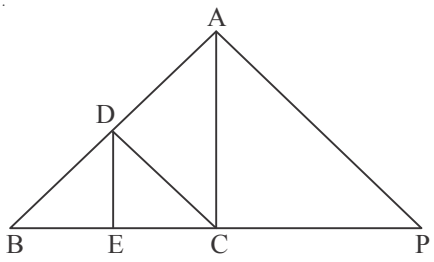
52. Two triangles BAC and BDC, right angled at A and D respectively are drawn on the same base BC and on the same side of BC. If AC and DB intersect at P. PRove that $AP \times PC = DP \times PB$. **(CBSE 2019)**



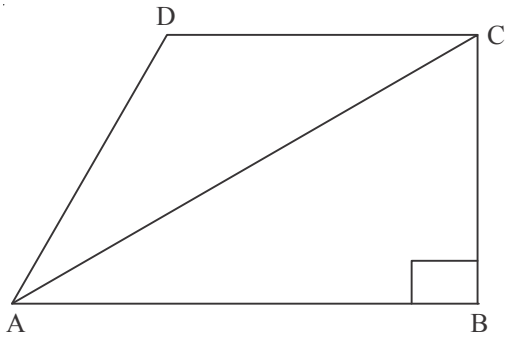
53. Hypotenuse of a right triangle is 25 cm and out of the remaining two sides, one is larger than the other by 5 cm, find the lenghts of the other two sides.

(NCERT Exempler)

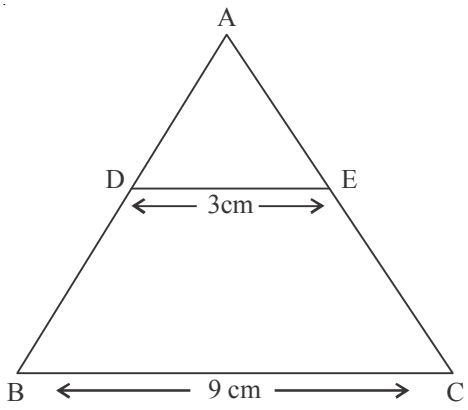
54. In the given figure DE || AC and $\frac{BE}{EC} = \frac{BC}{CP}$. Prove that DC || AP.



55. In a quadrilateral ABCD, $\angle B = 90°$, $AD^2 = AB^2 + BC^2 + CD^2$. Prove that $\angle ACD = 90°$.



56. In the given figure, DE || BC, DE = 3 cm, BC = 9 cm and ar (DADE) = 30 cm^2. Find ar (BCED).



57. In an equilateral ΔABC, D is a point on side BC such that $BD = \frac{1}{3}BC$. Prove

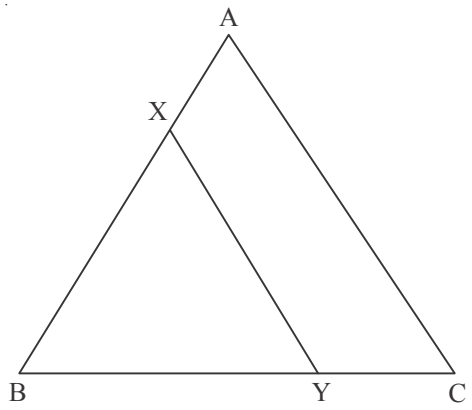
that $9AD^2 = 7AB^2$. **(NCERT, CBSE 2018)**

58. In ΔPQR, $PD \perp QR$ such that D lies on QR. If $PQ = a$, $PR = b$, $QD = c$ and $DR = d$ and a, b, c, d are positive units. Prove that $(a + b)(a - b) = (c + d)(c - d)$.

(NCERT Exempler)

59. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squars of their corresponding sides. **(CBSE 2010, 2018, 2019)**

60. In the given figure, the line segment XY is Parallel to AC of ΔABC and it divides the triangle into two parts of equal areas. Prove that $\frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$.



61. Through the vertex D of a parallelogram ABCD, a line is drawn to intersect the sides BA and BC produced at E and F respectively. Prove that $\frac{DA}{AE} = \frac{FB}{BE} = \frac{FC}{CD}$.

62. Prove that if in a triangle, the square on one side is equal to the sum of the squares on the other two sides, then the angle opposite to the first side is a right angle.

(CBSE 2019)

63. Prove that is a right angle triangle, the square of the hypotenuse is equal the sum of the squares of other two sides. **(CBSE 2018, 2019)**

64. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then prove that the other two sides are divided in the same ratio.

(CBSE 2019)

ANSWERS AND HINTS

VERY SHORT ANSWER TYPE QUESTIONS-I

1. (*i*) Similar (*ii*) $\frac{AB}{FE} = \frac{BC}{ED}$ (*iii*) Congruent

(*iv*) Same (*v*) Right

2. (*i*) False (*ii*) False (*iii*) True

(*iv*) True (*v*) True

3. (*a*) (*iii*) AAA similarity criterion.

(*b*) (*iv*) SSS similarity criterion.

(*c*) (*i*) SAS similarity criterion.

4. (B) $XY = \frac{1}{3}QR$

5. (A) 15 units

6. (C) Ratio of the square of their corresponding sides.

7. (C) 12 cm

8. (A) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

9. (B) 25 cm^2

10. (D) 90°

11. (B) 12 m

12. (B) 10 cm

13. (C) BC.DE = AB.EF

16. No, because $(12)^2 + (16)^2 \neq (18)^2$

17. 10 cm

18. $\Delta KPN \sim \Delta KLM$

$$\frac{x}{a} = \frac{c}{b+c}$$

$$x = \frac{ac}{b+c}$$

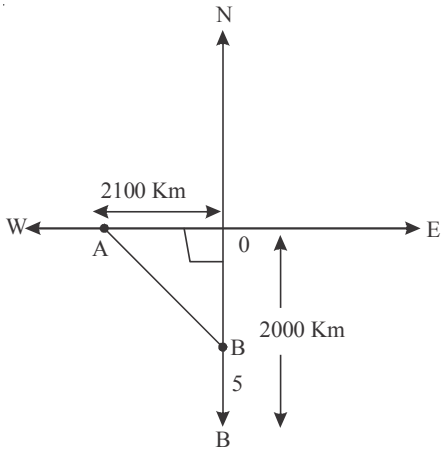
19. $\frac{AK}{AC} = \frac{HK}{BC} \Rightarrow \frac{10}{AC} = \frac{7}{3.5} \Rightarrow AC = 5 \text{ cm}$

20. $\angle D = \angle R$ (True)

$\angle F = \angle P$ (False)

21. 5 : 7

22.



$AB = \sqrt{(2100)^2 + (2000)^2} = 2900 \text{ km}$

23. Let longest side of the ΔDEF be x cm.

$\frac{225}{81} = \left(\frac{30}{x}\right)^2$

$x = 18$ cm

24. $\frac{AB}{PQ} = \frac{BC}{QR} \Rightarrow \frac{6}{4.5} = \frac{4}{x} \Rightarrow x = 3\text{cm}$

25. $\Delta PXY \sim \Delta PQR$

$\frac{PX}{PQ} = \frac{XY}{QR} = \frac{1}{3}$

$\therefore XY : QR = 1 : 3$

26. $\frac{x+3}{3x+19} = \frac{x}{3x+4}$ (By B.P.T.)

$x = 2$

27. $\frac{\text{ar (ABC)}}{\text{ar (DEF)}} = \left(\frac{BC}{EF}\right)^2 = \left(\frac{3EF}{EF}\right)^2 = \left(\frac{3}{1}\right)^2$

$$\frac{117}{ar(DEF)} = 9 \quad \Rightarrow \quad ar(DEF) = 13 \text{ cm}^2$$

28. $\angle F = \angle C = 56°$

29. 2 : 3

30. $PR = \sqrt{(6)^2 + (8)^2} = 10$ cm.

As $QR^2 = PQ^2 + PR^2$, therefore $\angle QPR = 90°$.

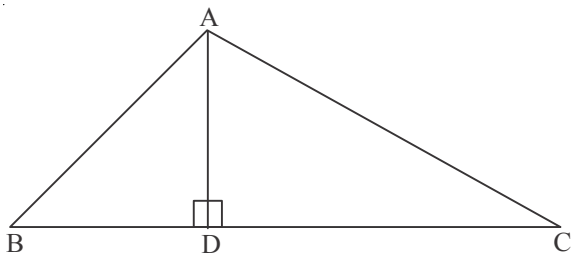
31. $DE \parallel AC, \frac{AD}{DB} = \frac{EC}{BE}$ ...(1) [$\because$ BPT]

$DF \parallel AE, \frac{AD}{DB} = \frac{FE}{BF}$ ...(2) [$\because$ BPT]

From (1) and (2), we get

$$\frac{FE}{BF} = \frac{EC}{BE}$$

32. In ΔADC, $AD^2 = AC^2 - DC^2$ ...(1)



In ΔADB, $AD^2 = AB^2 - BD^2$ ...(2)

Adding (1) and (2), we have

$2AD^2 = AC^2 + AB^2 - BD^2 - DC^2$

$2AD^2 + BD^2 + DC^2 = AC^2 + AB^2$

$2BD \times CD + BD^2 + DC^2 = AC^2 + AB^2$

$(BD \times DC)^2 = AC^2 + AB^2$

$BC^2 = AC^2 + AB^2$

By converse of Pythagoras Theorem, ΔABC is a right angled triangle.

33. $\angle B = \angle AED$ (Given)

$\angle A = \angle A$ (Common)

$\therefore \Delta ABC \sim \Delta AED$ [AA similarity criterion]

34. $\frac{3x-1}{5x-3} = \frac{2x+1}{6x-5} \Rightarrow x = \frac{1}{2}$ or 2

But $x = \frac{1}{2}$ is neglected due $(5x-3)$ get negative value.

So, $x = 2$ is the required value.

35. $\frac{PX}{XQ} = \frac{PY}{YR} \Rightarrow \frac{1}{2} = \frac{4}{YR} \Rightarrow$ YR = 8 cm

$\therefore$ PR = 8 + 4 = 12 cm

QR = $\sqrt{(12)^2 - (6)^2} = 6\sqrt{3}$ cm

36. $\Delta ABC \sim \Delta EDC$ (AA Similarity criterion)

$\frac{6}{3} = \frac{5}{CD}$

CD = 2.5 cm

37. $\Delta BOE \sim \Delta DOA$ (AA Similarity criterion)

$\frac{BO}{DO} = \frac{BE}{DA}$

$\frac{1}{2} = \frac{1.5}{DA}$

DA = 3 cm

BC = DA = 3 cm (Opposite sides of a parallelogram)

38. (*i*) 65°

(*ii*) 45°

(*iii*) 45°

(*iv*) 70°

39. $\frac{\text{ar}(\Delta ABC)}{\text{ar}(\Delta PQR)} = \left(\frac{144}{96}\right)^2 = \frac{9}{4}$

$\therefore$ ar (ΔABC) : ar $(\Delta PQR) = 9 : 4$

40. In ΔPQR, $\angle 1 = \angle 2$

$PR = PQ$ [Opposite sides of equal angles]

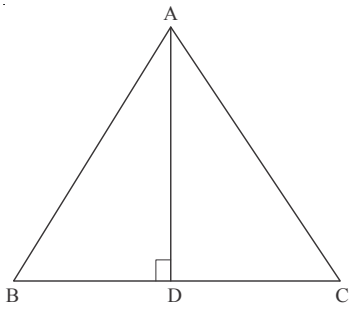
$\therefore \frac{QR}{QS} = \frac{QT}{PQ}$ and $\angle 1 = \angle 1$ (Common)

$\therefore \Delta PQS \sim \Delta TQR$ (SAS Similarity criterion)

41. $\Delta ADB \cong \Delta ADC$

$BD = DC$

$\therefore BD = \frac{1}{2} BC$ ...(1)



In right angled ΔADB,

$AB^2 = AD^2 + BD^2$

$BC^2 = AD^2 + \left(\frac{BC}{2}\right)^2$ [$\because$ AB = BC = CA and from (1)]

$3BC^2 = 4AD^2$

42. $\Delta ABC \sim \Delta CBD$

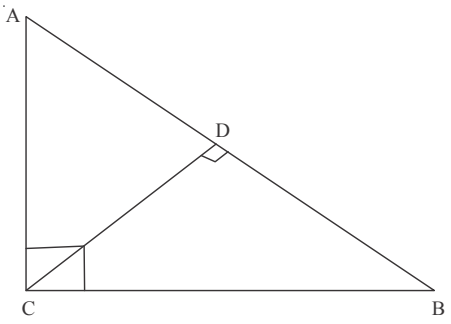
$\therefore BC^2 = AB.BD$ ...(1)

$\Delta ABC \sim \Delta ACD$

$\therefore AC^2 = AB.AD$ ...(2)

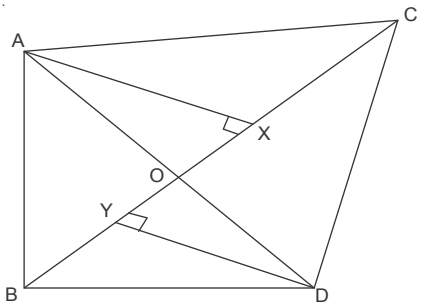
Divide (1) by (2), we get

$\frac{BC^2}{AC^2} = \frac{BD}{AD}$



43. Draw $AX \perp BC$ and $DY \perp BC$

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} \times BC \times AX}{\frac{1}{2} \times BC \times DY} = \frac{AX}{DY} \quad ...(1)$$



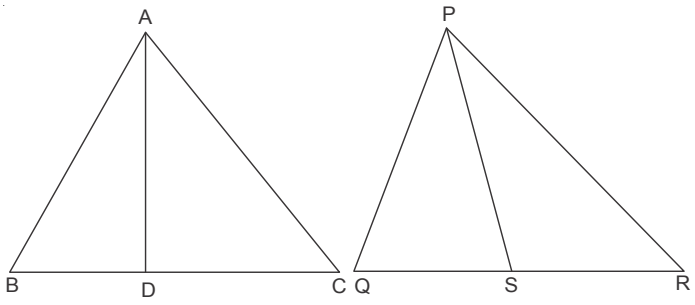
$\Delta AXO \sim \Delta DYO$ [AA similarity criterion]

$\frac{AX}{DY} = \frac{AO}{DO}$ ...(2) (C.P.S.T.)

From (1) and (2), we get

$\frac{ar\,(\Delta ABC)}{ar\,(\Delta DBC)} = \frac{AO}{DO}$

44.



As $\Delta ABC \sim \Delta PQR$, Hence $\angle B = \angle Q$ and $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{BD}{QS}$

In ΔABD and ΔPQS

$\frac{AB}{PQ} = \frac{BD}{QS}$ and $\angle B = \angle Q$.

$\therefore \Delta ABD \sim \Delta PQS$ (SAS Similarity criterion).

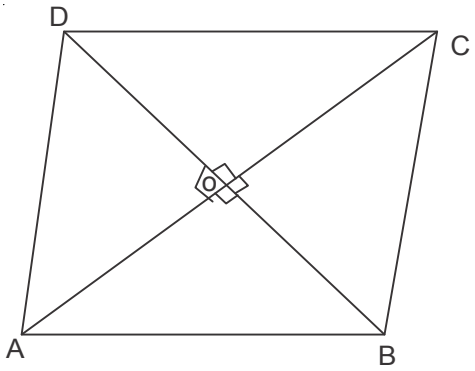
Hence, $\frac{AB}{PQ} = \frac{AD}{PS}$ (C.P.S.T.)

45. $\Delta BED \sim \Delta BCA$

$$\frac{x}{y} = \frac{a}{a+b}$$

$$\Rightarrow x = \frac{ay}{a+b}$$

46.



In right angled ΔAOB, $AB^2 = OA^2 + OB^2$ ...(1)

In right angled ΔBOC, $BC^2 = OB^2 + OC^2$ ...(2)

In right angled ΔCOD, $CD^2 = OC^2 + OD^2$ ...(3)

In right angled ΔDOA, $DA^2 = OD^2 + OA^2$ ...(4)

Adding (1), (2), (3) and (4), we get

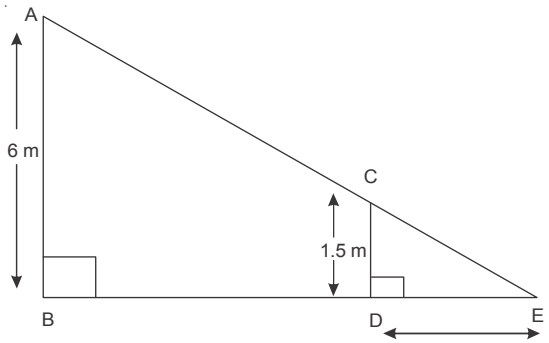
$$AB^2 + BC^2 + CD^2 + DA^2 = 2OA^2 + 2OB^2 + 2OC^2 + 2OD^2$$

$$= 2\left(\frac{1}{2}AC\right)^2 + 2\left(\frac{1}{2}BD\right)^2 + 2\left(\frac{1}{2}AC\right)^2 + 2\left(\frac{1}{2}BD\right)^2$$

[$\because$ Diagonals of rhombus $\perp$ bisect each other]

$$= AC^2 + BD^2$$

47. $\Delta ABE \sim \Delta CDE$

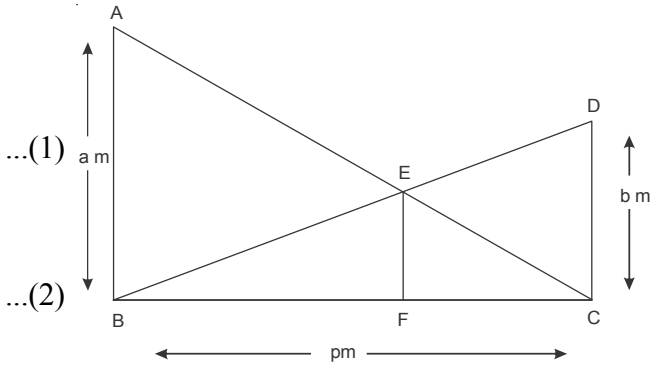


$$\frac{AB}{CD} = \frac{BE}{DE}$$

$$\frac{6}{1.5} = \frac{3+BD}{3}$$

$$BD = 9m$$

48. **To prove :** $EF = \frac{ab}{a+b}$



Proof : $AB \parallel EF \parallel DC$

$\Delta EFC \sim \Delta ABC$

$$\frac{EF}{AB} = \frac{FC}{BC}$$

$\Delta BFE \sim \Delta BCD$

$$\frac{EF}{CD} = \frac{BF}{BC}$$

Adding (1) and (2), we get

$$\frac{EF}{AB} + \frac{EF}{CD} = \frac{FC+BF}{BC}$$

$$EF\left[\frac{1}{AB} + \frac{1}{CD}\right] = \frac{BC}{BC}$$

$$EF\left[\frac{1}{a} + \frac{1}{b}\right] = 1$$

$$EF = \frac{ab}{a+b}$$

49. Same as Q. 48.

50. $\frac{PS}{SQ} = \frac{PT}{TR}$

By converse of BPT, $ST \parallel QR$

$\therefore \quad \angle PQR = \angle PST$ (A.I.A)

But $\angle PST = \angle PRQ$

So, $\angle PQR = \angle PRQ$

$\therefore \quad PQ = PR$

So, ΔPQR is an isosceles triangle.

51. In ΔOAB, $\frac{OD}{DA} = \frac{OE}{EB}$ .... (1) ($\because$ BPT)

In ΔOBC, $\frac{OE}{EB} = \frac{OF}{FC}$ .... (2) ($\because$ BPT)

From (1) and (2), we get

$$\frac{OD}{DA} = \frac{OF}{FC}$$

By converse of BPT, DF || AC.

52. $\Delta APB \sim \Delta DPC$ (AA Similarity criterion)

$$\frac{AP}{DP} = \frac{PB}{PC}$$ ($\because$ C.P.S.T.)

$AP.PC = DP.PB$

53. Let sides of right angled triangle other than hypotenuse be x cm and $(x + 5)$ cm.

By Pythagoras theorem,

$(x)^2 + (x + 5)^2 = (25)^2$

$x = 15$ or -20

But side is always positive, So, $x = 15$.

$\therefore$ Length of two sides is 15 cm and 20 cm.

54. Same as Q.31.

55. In right angled ΔABC, $AC^2 = AB^2 + BC^2$ ...(1)

Given, $AD^2 = (AB^2 + BC^2) + CD^2$

$\Rightarrow \quad AD^2 = AC^2 + CD^2$ [From (1)]

By converse of Pythagoras theorem, $\angle ACD = 90°$.

56. $\Delta ADE \sim \Delta ABC$

$$\frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \left(\frac{DE}{BC}\right)^2$$

$$\frac{30}{ar(\Delta ABC)} = \left(\frac{3}{9}\right)^2$$

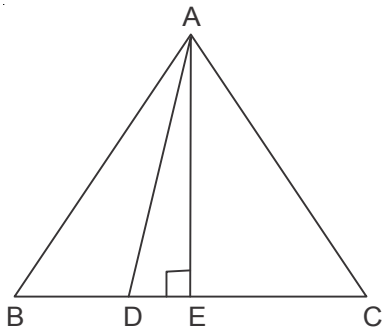
$\therefore$ ar(ΔABC) = 270 cm^2

ar (BCFD) = ar (ΔABC) – ar (ΔADE)

= 270 – 30 = 240 cm^2

57. Draw AE $\perp$ BC

ΔABE $\cong$ ΔACE



$$\therefore BE = CE \Rightarrow BE = \frac{1}{2}BC$$

In right angled ΔAED, $AE^2 = AD^2 - DE^2$ ...(1)

In right angled ΔAEB, $AE^2 = AB^2 - BE^2$ ...(2)

From (1) and (2), we have

$AD^2 - DE^2 = AB^2 - BE^2$

$$AD^2 - (BE - BD)^2 = BC^2 - \left(\frac{1}{2}BC\right)^2$$

$$AD^2 - \left[\frac{1}{2}BC - \frac{1}{3}BC\right]^2 = BC^2 - \frac{BC^2}{4}$$

$9AD^2 = 7AB^2$

58. In right angled ΔPDQ,

$PD^2 = a^2 - c^2$ ...(1)

In right angled ΔPDR

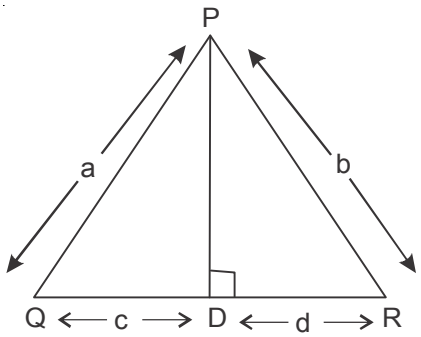
$PD^2 = b^2 - d^2$ ...(2)

From (1) and (2), we have

$a^2 - c^2 = b^2 - d^2$

$a^2 - b^2 = c^2 - d^2$

$(a - b)(a + b) = (c + d)(c - d)$



59. Theorem 6.6 of NCERT.

60. Given, ar ΔBXY = ar AXYC

ar (Δ ABC) = ar Δ BXY + ar AXYC

= 2 ar ΔBXY

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta BXY)} = \frac{2}{1}$$

$\Delta ABC \sim \Delta XBY$

$$\left(\frac{AB}{XB}\right)^2 = \frac{\text{ar}\,(\Delta ABC)}{\text{ar}\,(\Delta BXY)}$$

$$\frac{AB}{XB} = \sqrt{2}$$

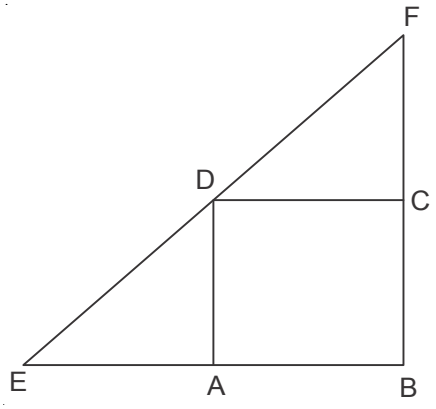
$$\frac{XB}{AB} = \frac{1}{\sqrt{2}}$$

$$1 - \frac{XB}{AB} = 1 - \frac{1}{\sqrt{2}}$$

$$\frac{AB - XB}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

$$\frac{AX}{AB} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

61. $\Delta EAD \sim \Delta EBF$



$$\frac{EA}{EB} = \frac{AD}{BF}$$

$$\Rightarrow \frac{BF}{BE} = \frac{AD}{AE} = \frac{BF - AD}{BE - AE} = \frac{BF - BC}{BA} = \frac{CF}{DC}$$

62. Theorem 6.9 of NCERT.

63. Theorem 6.8 of NCERT.

64. Theorem 6.9 of NCERT.

PRACTICE-TEST

Triangles

Time : 1 Hrs. ***M.M. : 20***

SECTION - A

1. If sides of two similar triangles are in the ratio of 8:10, then areas of these triangles are in the ratio _____ . **1**

2. If in two triangles ΔABC and ΔPQR, $\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$, then **1**

(A) $\Delta PQR \sim \Delta CAB$ (B) $\Delta PQR \sim \Delta ABC$

(C) $\Delta CBA \sim \Delta PQR$ (D) $\Delta BCA \sim \Delta PQR$

3. ΔABC is an isosceles right triangle, right angled at C, then AB^2 = ...

(A) AC^2 (B) $2\,AC^2$

(C) $4\,AC^2$ (D) $3\,AC^2$ **1**

4. A line DE is drawn parallel to base BC of ΔABC, meeting AB in D and AC at E. If $\frac{AB}{BD} = 4$ and CE = 2 cm, find the length of AE.

SECTION B

5. The length of the diagonal of a rhombus field are 32 m and 24 m. Find the length of the side of the field. **2**

6. A man goes 24 m towards West and then 10 m towards North. How far is he from the starting point? **2**

7. Using converse of Basic Proportionality Theorem, prove that the line joining the mid-points of any two sides of a triangle is parallel to the third side. **2**

SECTION C

8. E is a point on the side AD produced of a parallelogram ABCD and BE intersect CD at F. Show that $\Delta ABE \sim \Delta DCB$. **3**

9. In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitude. **3**

SECTION D

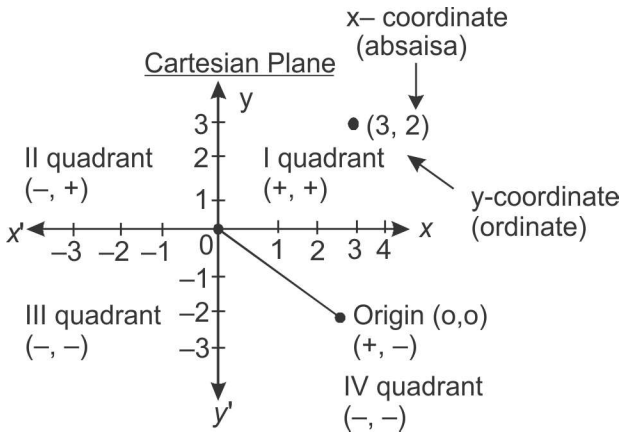
10. State and prove Basic Proportionality Theorem. **4**

CHAPTER

7 Co-ordinate Geometry

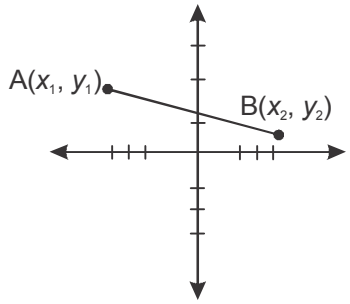
Key Points

1. The system of geometry where the position of points on the plane is described using an ordered pair of numbers.



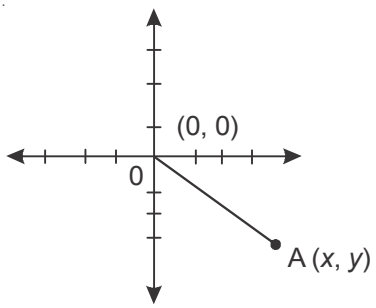
2. Distance Formula

Finding distance betwen tow given points :



AB (Distance between A and B) = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance of a point from origin :

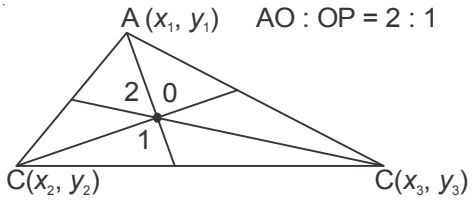


OA = $\sqrt{x^2 + y^2}$

Midpoint formula :

Cordinates of mid points of AB where $A(x_1, y_1)$ and $B(x_2, y_2)$ are : $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Centroid of a triangle is given by :

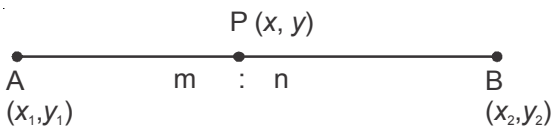


$O\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

Section formula:

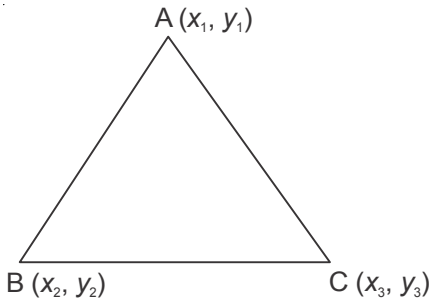
The coordinates of a point $P(x, y)$ which divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ are given by

$P\left(x = \frac{mx_2 + nx_1}{m + n}, y = \frac{my_2 + ny_1}{m + n}\right)$



The area of triangle ABC

$$= \frac{1}{2}\,[x_1\,(y_2 - y_3) + x_2\,(y_3 - y_1) + x_3\,(y_1 - y_2)] \text{ sq. units}$$



If area of a triangle is zero then points are collinear.

VERY SHORT ANSWER TYPE QUESTIONS

Fill in the blanks :

1. The distance of a point from the y-axis is called its x-coordinate or _____ .

2. The distance of a point from the x-axis is called its _____ or ordinate.

3. The point (5, 0) lies on ____ axis.

4. A point which lies on y-axis are of the form _____ .

5. A linear equation of the form $ax + by + c = 0$ when represented graphically gives a _____ .

6. The distance of a point $P(x, y)$ from the origin is _____

Multiple Choice Question :

7. P is a point on x-axis at a distance of 3 unit from y-axis to its left. The co-ordinates of P are :

(a) (3, 0) (b) (0, 3)

(c) (– 3, 0) (d) (0, – 3)

8. The distance of $P(3, -2)$ from y-axis is

(a) 3 units (b) 2 units

(c) – 2 units (d) $\sqrt{13}$ units

9. The co-ordinates of two ponts are (6, 0) and (0, – 8). The co-ordinates of the mid points are

(a) 3, 4 (b) 3, – 4

(c) 0, 0 (d) – 4, 3

10. If the distance between $P(4, 0)$ and $Q(0, x)$ is 5 units, the value of x will be

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

11. The co-ordinates of the point where line $\frac{x}{a}+\frac{y}{b}=7$ intersects *y*-axis are

(*a*) *a*, 0 (*b*) 0, *b*

(*c*) 0, 7*b* (*d*) 2*a*, 0

12. The area of triangle OAB, the co-ordinates of whose vertices are A(4, 0), B(0, – 7) and *O* origin, is :

(*a*) 11 sq. units (*b*) 18 sq. units

(*c*) 28 sq. units (*d*) 14 sq. units

13. The distance between the points $P\left(-\frac{11}{3}, 5\right)$ and $Q\left(-\frac{2}{3}, 5\right)$ is

(*a*) 6 units (*b*) 4 units

(*c*) 3 units (*d*) 2 units

14. The distance between the points (5 cos 35°, 0) and (0, 5 cos 55°) is

(*a*) 10 units (*b*) 5 units

(*c*) 1 unit (*d*) 2 units

15. The co-ordinates of vertex A of ΔABC are (– 4, 2) and a point D which is mid point of BC are (2, 5). The coordinates of centroid of ΔABC are

(*a*) (0, 4) (*b*) $\left(-1, \frac{7}{2}\right)$

(*c*) $\left(-2, \frac{7}{3}\right)$ (*d*) (0, 2)

16. The distance between the line $2x + 4 = 0$ and $x - 5 = 0$ is

(*a*) 9 units (*b*) 1 unit

(*c*) 5 units (*d*) 7 units

17. The perimeter of triangle formed by the points (0, 0), (2, 0) and (0, 2) is

(*a*) 4 units (*b*) 6 units

(*c*) $6\sqrt{2}$ units (*d*) $4+2\sqrt{2}$ units

18. If the centroid of the triangle formed by (9, *a*), (*b*, – 4) and (7, 8) is (6, 8), then the value *a* and *b* are :

(*a*) $a = 4, b = 5$ (*b*) $a = 5, b = 4$

(*c*) $a = 5, b = 2$ (*d*) $a = 3, b = 2$

State True or False

19. The point $P(-4, 2)$ lies on the line segment joining the points A$(-4, 6)$ and B$(-4, -6)$

20. The points $(0, 5)$, $(0, -9)$ and $(3, 6)$ are collinear.

21. For what value of P, the points $(2, 1)$, $(p, -1)$ and $(-1, 3)$ are collinear.

22. Find the area of ΔPQR, whose vertices are $P(-5, 7)$, $Q(-4, -5)$ and $R(4, 5)$.

23. Find the point of trisection of the linear segment joining the points $(1, -2)$ and $(-3, 4)$.

24. The midpoints of the sides of a triangle are $(3, 4)$, $(4, 1)$ and $(2, 0)$. Find the vertices of the triangle.

25. Find the value of x if the points A$(4, 3)$ and B$(x, 5)$ lie on a circle whose centre is $O(2, 3)$.

26. Find the ratio in which x-axis divides the line segment joining the points $(6, 4)$ and $(1, -7)$.

27. Show that the points $(-2, 3)$, $(8, 3)$ and $(6, 7)$ are the vertices of a right angle triangle.

28. Find the point on the y-axis which is equidistant from the points $(5, -2)$ and $(-3, 2)$.

29. Find the ratio in which y-axis divides the line segment joining the points A$(5, -6)$ and B$(-1, -4)$.

30. Find the co-ordinates of a centroid of a triangle whose vertices are $(3, -5)$, $(-7, 4)$ and $(10, -2)$.

31. Find the relation between x and y such that the points (x, y) is equidistant from the points $(7, 1)$ and $(3, 5)$.

32. Find the ratio in which the line segment joining the points $(1, -3)$ and $(4, 5)$ is divided by x-axis. Also find the co-ordinates of this point on x-axis.

33. What is the value of a if the points $(3, 5)$ and $(7, 1)$ are equidistant from the point $(a, 0)$?

34. Find a relation between x and y if the prints $A(x, y)$, $B(-4, 6)$ and $C(-2, 3)$ are collinear.

35. Find the area of a triangle whose vertices are given as $(1, -1)$, $(-4, 6)$ and $(-3, -5)$.

36. Name the type of triangle formed by the points A(– 5, 6), B(– 4, – 2) and C(7, 5).
(NCERT Exempler)

37. Find the points on the x-axis which are at a distance of $2\sqrt{5}$ from the point (7, – 4). How many such points are there? **(NCERT Exempler)**

38. What type of quadrilateral do the points $A(2, -2)$, $B(7, 3)$, $C(11, -1)$ and $D(6, -6)$, taken in that order, form ?

39. Find the co-ordinates of the point Q on the x-axis which lies on the perpendicular bisector of the line-segment joining the points $A(-5, -2)$ and $B(4, -2)$. Name the type of triangle formed by the points Q, A and B.

40. Let P and Q be the points of trisection of the line segment joining the points $A(2, -2)$ and B(– 7, 4) such that P is nearer to A. Find the co-ordinates of P and Q.

SHORT ANSWER TYPE QUESTIONS-II

41. The line segment joining the points $A(2, 1)$ and $B(5, -8)$ is trisected at the point P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of k.

42. Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points $(-2, -5)$ and $(6, 3)$. Find the co-ordinates of the point of intersection.
HOTS

43. Point A lies on the line segment XY joining X(6, – 6) and Y(– 4, – 1) in such a way that $\frac{XA}{XY} = \frac{2}{5}$. If point A also lies on the line $3x + k(y + 1) = 0$, find the value of k.

HOTS

44. Find the area of the triangle formed by joining the mid points of the sides of the triangle ABC, whose vertices are $A(0, -1)$, $B(2, 1)$ and $C(0, 3)$.

45. Find the value of k so that the area of triangle ABC with $A(k + 1, 1)$, $B(4, -3)$ and $C(7, -k)$ is 6 square units.

46. Point P divides the line segment joining the points $A(2, 1)$ and $B(5, -8)$ such that

$\frac{AP}{PB} = \frac{1}{3}$. If P lies on the line $2x - y + k = 0$. Find the value of k.

47. A point P on the x-axis divides the line segment joining the points $(4, 5)$ and $(1, -3)$ in certain ratio. Find the co-ordinates of point P.

48. In right angled ΔABC, $\angle B = 90°$ and $AB = \sqrt{34}$ units. The co-ordinates of points B, C are $(4, 2)$ and $(-1, y)$ respectively. If ar $\Delta ABC = 17$ sq. units, then find the value of y.

49. If $A(-3, 2)$, $B(x, y)$ and $C(1, 4)$ are the vertices of an isosceles triangle with $AB = BC$. Find the value of $(2x + y)$.

50. If the point $P(3, 4)$ is equidistant from the points $A(a + b, b - a)$ and $B(a - b, a + b)$ then prove that $3b - 4a = 0$.

LONG ANSWER TYPE QUESTIONS-III

51. If $A(-5, 7)$, $B(-4, -5)$, $C(-1, -6)$ and $D(4, 5)$ are vertices of a quadrilateral ABCD. Find the area of quadrilateral ABCD.

52. If $P(x, y)$ is any point on the line joining $A(a, 0)$ and $B(0, b)$ then show that $\frac{x}{a} + \frac{y}{b} = 1$.

53. If the points (x, y), $(-5, -2)$ and $(3, -5)$ are collinear, prove that $3x + 8y + 31 = 0$.

54. Find the relation between x and y if $A(x, y)$, $B(-2, 3)$ and $C(2, 1)$ form an isosceles triangle with $AB = AC$.

55. Prove that the point $\left(x, \sqrt{1 - x^2}\right)$ is at a distance of 1 unit from the origin.

56. If $R(x, y)$ is point on the line segment joining the points $A(a, b)$ and $B(b, a)$, then prove that $x + y = a + b$.

57. If the points (a, b), (c, d) and $(a - c, b - d)$ are collinear show that $bc = ad$.

58. Find the co-ordinates of the circumcenter of the triangle whose vertices are (3, 7), (0, 6) and (– 1, 5). Find the circumradius. **(HOTS)**

59. In a triangle PQR, the co-ordinates of points P, Q and R are (3, 2), (6, 4) and (9, 3) respectively. Find the co-ordinates of centroid G. Also find the areas of ΔPQG and ΔPRG.

60. If the points (5, 4) and (x, y) are equidistant from the point (4, 5), prove that $x^2 + y^2 - 8x - 10y + 39 = 0$.

VERY SHORT ANSWER TYPE QUESTIONS-I

1. abscissa

2. y-coordinate

3. x-axis

4. $(0, y)$

5. straight line

6. $\sqrt{x^2 + y^2}$

7. (*iii*) $(-3, 0)$

8. (i) 3 units

9. (ii) $(3, -4)$

10. (ii) 3

11. (iii) $(0, 7b)$

12. (iv) 14 sq. units

13. (c) 3 units

14. (b) 5 units

15. (a) $(0, 4)$

16. (d) 7 units

17. (d) $\left(4 + 2\sqrt{2}\right)$ units

18. (d) a = 20, $b = 2$

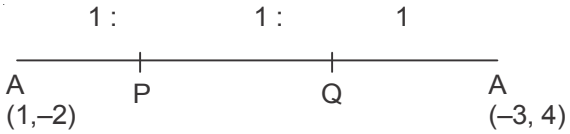
19. False

20. False

21. $P = 3$

22. 25 sq. units

23.



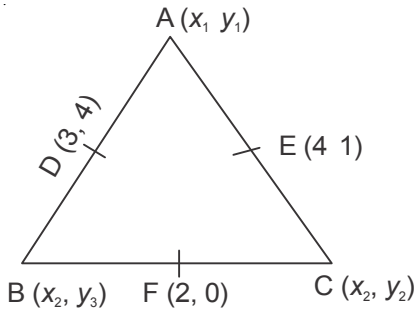
AP : PB = 1 : 2

AQ : QB = 2 : 1

$P = \left(-\frac{1}{3}, 0\right)$

$Q = \left(-\frac{5}{3}, 2\right)$

24.



$\frac{x_1 + x_3}{2} = 3,$ $\frac{y_1 + y_3}{2} = 4$

$x_1 + x_3 = 6$ $y_1 + y_3 = 8$

Similarly,

$x_1 + x_2 = 8$ $y_1 + y_3$

$x_2 + x_3 = 4$

$x_3 = 1, \quad x_2 = 3, \quad x_1 = 5$

$A(1, 3), B(5, 5), C(3, -3)$

25. $x = 0$

26. 4 : 7

27. Show using pythagoras and distance formula.

28. $y = 4$

29. 5 : 1

30. $(2, -1)$

31. $x - y = 2$

32. 3 : 5 ; $\left(\frac{17}{8}, 0\right)$

33. $a = 2$

34. $3x = -2y$

35. 28 sq. units.

36. Using distance formula, scalene triangle.

37. $x = 1, x = -15$

Two such points are there.

38. Rhombus.

39. Use distance formula and midpoint formula.

$Q\left(-\frac{1}{2}, 0\right)$

Δ is isosceles.

40. $P(-1, 0)$, $Q(-4, 2)$

41. $P(3, -2)$

Put value of $x = 3, y = -2$ is equation, then $k = -8$.

42. Let $P(x, y)$ be the point and $m : n$ is the ratio

then $x = \frac{6n - 2m}{m + n}$, $y = \frac{3n - 5m}{m + n}$ ...(1)

From equation of line $x = 3y \Rightarrow \frac{x}{y} = 3$

By putting $x = 3y$ or $\frac{x}{y} = 3$ is (1)

$m : n = 3 : 13$

Then $P(x, y) = \left(\frac{9}{2}, \frac{3}{2}\right)$

43. Find $\frac{XA}{AY} = \frac{2}{3}$.

Let $A(x, y)$ is the point.

$x = 2, y = -4$

$A(2, -4)$

Put $x = 2$ and $y = -4$ in equation.

$\therefore K = 2$

44. 1 sq. unit

45. $K = 3$

46. $K = \frac{-17}{4}$

47. $m : n = 5 : 3$

$P\left(\frac{17}{8}, 0\right)$

48. $y = -1, y = 5$

49. $2x + y = 1$

50. $3b - 4a = 0$ proved by using distance formula.

51. Area of quadrilateral ABCD = Area of ΔABC + Area of ΔADC

Ar (ABCD) = 72 sq. units.

52. Prove by section formula.

53. Prove by area of $\Delta = 0$ if points are collinear.

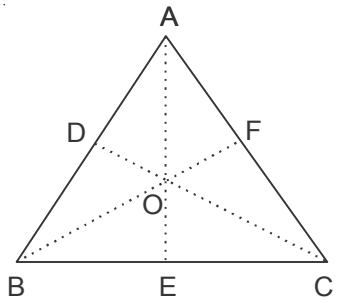
54. Prove by distance formula.

55. Prove by distance formula.

56. Prove by using area of triangle = 0 if points are collinear.

58. Find co-ordinates of mid points of AB, BC, CA

then DO = OE = OF



then (circumcentre) $O(x, y) = \left(1, \frac{13}{2}\right)$

circumradius AO = $\frac{\sqrt{17}}{2}$.

59. $G(x, y) = (6, 3)$

ar Δ PQG = $\frac{3}{2}$ sq. units

ar Δ PRG = $\frac{3}{2}$ sq. units

60. Use distance formula

PRACTICE-TEST

Coordinate Geometry

Time : 1 Hr. ***M.M. : 20***

SECTION - A

1. Find the value of m in which the points (3, 5), (m, 6) and $\left(\frac{1}{2}, \frac{15}{2}\right)$ are collinear. **1**

2. What is the distance between the points A(c, 0) and B(0, $-c$) **1**

3. The distance of point P(– 6, 8) from the origin is _____ . **1**

4. Find the value of 'a' so that the point (3, a) lies on the line segment $2x - 3y = 5$. **1**

SECTION B

5. For what value of p, the points (– 3, 9), (2, p) and (4, – 5) are collinear. **2**

6. If the points A(8, 6) and B(x, 10) lie on the circle whose centre is (4, 6) then find the value of x. **2**

7. Find the perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0). **2**

SECTION C

8. Show that the points A(–3, 2), B(– 5, –5), C(2, –3) and D(4, 4) are the vertices of a rhombus. **3**

9. Find the ratio in which the point (2, y) divides the line segment joining the points A(– 2, 2) and B(3, 7). Also find the value of y. **3**

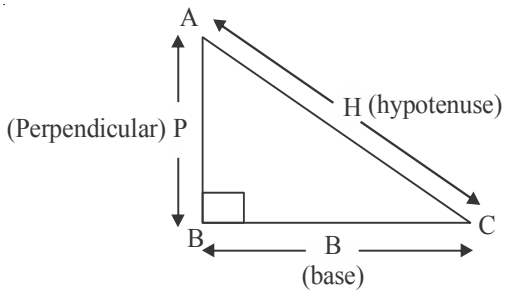
SECTION D

10. If the point P divides the line segment joining the points A(–2, –2) and B(2, – 4) such that $\frac{AP}{AB} = \frac{3}{7}$, then fidn the coordinate of P. 4

CHAPTER

8 Introduction to Trigonometry

KEY POINTS

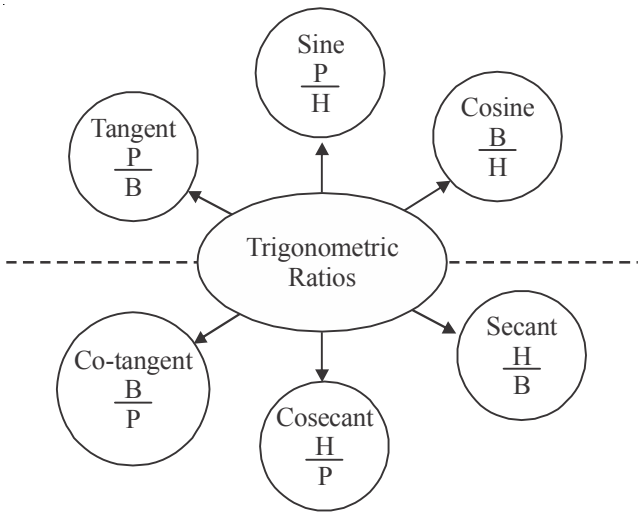


- A branch of mathematics which deals with the problems related to right angled triangles. It is the study of relationship between the sides and angles of a right angled triangel.

  Note : For ∠A — Perpendicular is BC base is AB.

  For ∠C, Perpendicualr is AB Base is BC.

Trigonometric Rations of an acute angle in a right angled triangle express the relationship between the angle and the length of its sides.

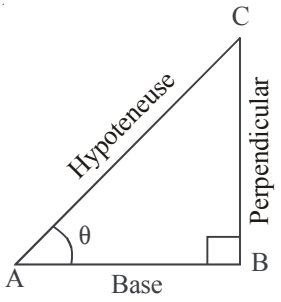


Mind Trick: To learn the relationship of sine, cosine and tangent follow this sentences.

Some People Have Curly Brown Hair Through Proper Brushing

$\sin A = \frac{P}{H}$ $\cos A = \frac{B}{H}$ $\tan A = \frac{P}{B}$

1. Trigonometric ratio : In ΔABC, $\angle B = 90°$. For $\angle A$,



$$\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\text{adjacent side}}{\text{Hypotenuse}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{\text{Opposite side}}{\text{adjacent side}}$$

$$\cot A = \frac{\text{Base}}{\text{Perpendicular}} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{\text{Hypotenuse}}{\text{adjacent side}}$$

$$\text{cosec } A = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{\text{Hypotenuse}}{\text{Opposite side}}$$

2. Opposites

$$\sin\theta = \frac{1}{\text{cosec }\theta}, \ \text{cosec }\theta = \frac{1}{\sin\theta}$$

$$\cos\theta = \frac{1}{\sec\theta}, \ \sec\theta = \frac{1}{\cos\theta}$$

$$\tan\theta = \frac{1}{\cot\theta}, \ \cot\theta = \frac{1}{\tan\theta}$$

3. $\tan\theta = \frac{\sin\theta}{\cos\theta}, \cot\theta = \frac{\cos\theta}{\sin\theta}$

4. Identities

$\sin^2\theta + \cos^2 = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$ and $\cos^2\theta = 1 - \sin^2\theta$

$1 + \tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta = \sec^2\theta - 1$ and $\sec^2\theta - \tan^2\theta = 1$

$1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \cot^2\theta = \text{cosec}^2\theta - 1$ and $\text{cosec}^2\theta - \cot^2\theta = 1$

5. Trigonometric ratios of some specific angles

$\angle A$	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin A$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos A$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan A$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\cot A$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0
$\sec A$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\operatorname{cosec} A$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1

6. Trigonometric ratios of complimentary angles

$\sin(90° - \theta) = \cos\theta$

$\cos(90° - \theta) = \text{Sin}\,\theta$

$\tan(90° - \theta) = \cot\theta$

$\cot(90° - \theta) = \tan\theta$

$\sec(90° - \theta) = \operatorname{cosec}\theta$

$\operatorname{cosec}(90° - \theta) = \sec\theta$

VERY SHORT ANSWER TYPE QUESTIONS

1. If $\text{Sin}\,\theta = \cos\theta$, find the value of θ

2. If $\tan\theta = \cot(30° + \theta)$, find the value of θ

3. If $\text{Sin}\,\theta = \cos(\theta - 6°)$, find the value of θ

4. If $\cos A = \frac{7}{25}$, find the value of $\tan A + \cot A$

5. If $\tan\theta = \frac{4}{3}$ then find the value of $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$

6. If $3x = \text{cosec}\ \theta$ and $\frac{3}{x} = \cot\ \theta$ then find $3\left(x^2 - \frac{1}{x^2}\right)$

7. If $x = a \sin\ \theta$ and $y = a \cos\ \theta$ then find the value of $x^2 + y^2$

8. Find the value of $\text{cosec}\ 70° - \sec\ 20°$

9. If $5x = \sec\ \theta$ and $\frac{5}{x} = \tan\ \theta$ then find the value of $5\left(x^2 - \frac{1}{x^2}\right)$

10. Find the value of $9\ \sec^2 A - 9\ \tan^2 A$

11. Express $\sec\ \theta$ in terms of $\cot\ \theta$

12. Find the value of $\cos\ \theta \cos\ (90° - \theta) - \sin\ \theta \sin\ (90° - \theta)$

13. If $\sin\ (20° + \theta) = \cos\ 30°$ then find the value of θ.

14. Find the value of $\frac{1 + \tan^2\ \theta}{1 + \cot^2\ \theta}$

15. Find the value of $\frac{\sin\ \theta}{\sqrt{1 - \sin^2\ \theta}}$

16. Given $\tan\ \theta = \frac{1}{\sqrt{3}}$, find the value of $\frac{\text{cosec}^2\theta - \sec^2\ \theta}{\text{cosec}^2\theta + \sec^2\ \theta}$. (CBSE, 2010)

17. If $\theta = 45°$, then find the vlaue of $\text{cosec}^2\ \theta$. (CBSE, 2010)

18. If $\cos\ \theta = \frac{2}{3}$, then find the value of $2\ \sec^2\ \theta + 2\ \tan^2\ \theta - 7$. (CBSE, 2011)

19. Find the value of $6\ \tan^2\ \theta - 6\ \sec^2\ \theta$

20. Express $\text{cosec}\ 48° + \tan\ 88°$ in terms of trigonometric ratios of angle between $0°$ and $45°$.

21. If $5\ \tan\ \theta - 4 = 0$, then value of $\frac{5\ \sin\ \theta - 4\ \sin\ \theta}{5\ \sin\ \theta + 4\ \cos\ \theta}$ is

(a) $\frac{5}{3}$ (b) $\frac{5}{6}$ (c) 0 (d) $\frac{1}{6}$

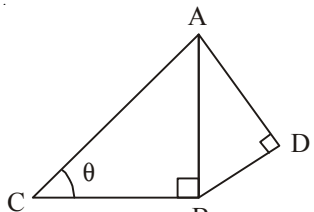
22. If A and B are complementary angles, then

(a) $\sin A = \sin B$ (b) $\cos A = \cos B$ (c) $\tan A = \tan B$ (d) $\sec A = \operatorname{cosec} B$

23. In Fig. if $AD = 4$ cm, BD = 3 cm and CB = 12 cm. then $\cot \theta =$

(a) $\frac{12}{5}$ (b) $\frac{5}{12}$

(c) $\frac{13}{12}$ (d) $\frac{12}{13}$



24. The value of tan 1°, tan 2°, tan 3° _____ tan 89° is.

(a) 1 (b) – 1 (c) 0 (d) None of these

25. If θ and $2\theta - 45°$ are acute angles such that $\sin \theta = \cos (2\theta - 45°)$ then $\tan \theta$ is

(a) 1 (b) – 1 (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$

SHORT ANSWER TYPE (I) QUESTIONS

Prove that :

26. $\sec^4 \theta - \sec^2 \theta = \tan^4 \theta + \tan^2 \theta$

27. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \tan\theta + \text{Sec}\,\theta$

28. If $x = p \sec\theta + q \tan\theta$ & $y = p \tan\theta + q \sec\theta$ then prove that $x^2 - y^2 = p^2 - q^2$

29. If $7 \sin^2\theta + 3 \cos^2\theta = 4$ then show that $\tan\theta = \frac{1}{\sqrt{3}}$

30. If $\text{Sin}\,(A - B) = \frac{1}{2}$, $\cos(A + B) = \frac{1}{2}$ then find the value of A and B.

31. Find the value of $\frac{\cos^2 20° + \cos^2 70°}{\sin^2 59° + \sin^2 31°}$.

32. **Prove that :** $\tan 1° \tan 11° \tan 21° \tan 69° \tan 79° \tan 89° = 1$

33. If $\sec 4A = \operatorname{cosec}(A - 20°)$ then find the value of A.

34. If $3 \cot A = 4$, find the value of $\frac{\text{Cosec}^2 A + 1}{\text{Cosec}^2 A - 1}$.

35. If $\tan (3x - 15°) = 1$ then find the value of x.

36. If A, B, C are interior angles of ΔABC, the prove that $\cos ec\left(\frac{A+B}{2}\right) = \sec\left(\frac{C}{2}\right)$.

(CBSE 2011)

37. In ΔABC, right angled at B, AB = 5 cm and $\angle ACB = 30°$. Find BC and AC.

38. If $\tan \theta = \cot (30° + \theta)$, Find the value of θ. (CBSE, 2012)

39. Show that : $\frac{1 - \sin 60°}{\cos 60°} = 2 - \sqrt{3}$. (CBSE, 2014)

40. Find the value of θ, if $\frac{\cos \theta}{1 - \sin \theta} + \frac{\cos \theta}{1 - \sin \theta} = 4$, $\theta \leq 90°$. (CBSE, 2014)

SHORT ANSWER TYPE QUESTIONS

Prove that :

41. $\frac{\tan A + \text{Sec}\, A - 1}{\tan A - \text{Sec}\, A + 1} = \frac{1 + \text{Sin}\, A}{\text{Cos}\, A}$

42. $\frac{1}{\sec x - \tan x} - \frac{1}{\cos x} = \frac{1}{\cos x} - \frac{1}{\sec x + \tan x}$

43. $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta = \sec \theta \, \text{cosec}\, \theta + 1$

44. $(\sin \theta + \text{cosec}\, \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$

45. $\sec A (1 - \sin A) (\sec A + \tan A) = 1$

46. If $\tan \theta + \sin \theta = m$, $\tan \theta - \sin \theta = n$ then show that $m^2 - n^2 = 4\sqrt{mn}$.

47. If $\sec \theta = x + \frac{1}{4x}$, prove that $\sec \theta + \tan \theta = 2x$ or $\frac{1}{2x}$

48. If $\sin \theta + \sin^2 \theta = 1$, prove that $\cos^2 \theta + \cos^4 \theta = 1$

49. Without using trigonometric table, the value of
$\cot \theta \tan (90° - \theta) - \sec (90° - \theta) \text{cosec}\, \theta + \sin^2 65° + \sin^2 25° + \sqrt{3} \tan 5° \tan 85°$.

50. Prove that : $\frac{\cot(90°-\theta)}{\tan\theta}+\frac{\operatorname{cosec}(90°-\theta)\sin\theta}{\tan(90°-\theta)}=\sec^2\theta$

51. Find the value of :

$\frac{\cos 20°+\cos^2 70°}{\sec^2 50°-\cot^2 40}+2\operatorname{Cosec}^2 58°-2\operatorname{Cot}58°\tan 32°-4\tan 13°\tan 37°\tan 77°\tan 45°\tan 53°$.

52. If A, B, C are the angles of ΔABC then prove that $\operatorname{cosec}^2\left(\frac{B+C}{2}\right)-\tan^2\frac{A}{2}=1$

53. Find the value of $\sec^2 10°-\cot^2 80°+\frac{\sin 15°\cos 75°+\cos 15°\sin 75°}{\cos\theta\sin(90°-\theta)+\sin\theta\cos(90°-\theta)}$.

54. Prove that : $\frac{\tan\theta-\cot\theta}{\sin\theta\cos\theta}=\tan^2\theta-\cot^2\theta$. (CBSE 2012)

55. If $\cos\theta+\sin\theta=\sqrt{2}\cos q$, then show that $\cos\theta-\sin\theta=\sqrt{2}\sin\theta$.

56. Evaulate : $4-\frac{\sin 30°+\tan 45°-\operatorname{cosec}60°}{\sec 30°+\cos 60°+\cos 45°}$ (CBSE, 2012)

57. Prove that : $1-\frac{\sin A\sin(90°-A)}{\cot(90°-A)}=\sin^2 A$ (CBSE, 2012)

58. If $a\cos\theta=b\sin\theta=m$ and $a\sin\theta-b\cos\theta=n$ (CBSE, 2001 C)
Prove that : $a^2+b^2=m^2+n^2$

59. If $a\cos\theta-b\sin\theta=c$ prove that $a\sin\theta+b\cos\theta=\pm\sqrt{a^2+b^2-c^2}$.

(CBSE, 2001 C)

60. Without using trigonometric tablets, evaluate :

$\frac{\sec^2 54°-\cot^2 36°}{\operatorname{cosec}^2 57°-\tan^2 33°}+2\sin^2 38°\sec^2 52°-\sin^2 45°$ (CBSE, 2005)

LONG ANSWER TYPE QUESTIONS

Prove That:

61. $\frac{\operatorname{Sec}\theta+\tan\theta-1}{\tan\theta-\operatorname{Sec}\theta+1}=\frac{\operatorname{Cos}\theta}{1-\operatorname{Sin}\theta}$

62. $\left(1+\frac{1}{\tan^2\theta}\right)\left(1+\frac{1}{\text{Cot}^2\,\theta}\right)=\frac{1}{\text{Sin}^2\,\theta-\text{Sin}^4\,\theta}$

63. $2(\sin^6\theta+\cos^6\theta)-3(\sin^4\theta+\cos^4\theta)+1=0$

64. $(1+\cot A+\tan A)(\sin A-\cos A)=\sin A\tan A-\cot A\cos A$

65. If $\text{Sin}\,\theta+\text{Cos}\,\theta=m$ and $\text{Sec}\,\theta+\text{Cosec}\,\theta=n$ then show that $n(m^2-1)=2m$

66. find the value of :

$$\frac{\text{Cot}\,(90°-\theta)\tan\theta-\text{Cosec}\,(90°-\theta)\text{Sec}\,\theta}{\text{Sin}\,12°\,\text{Cos}\,15°\,\text{Sec}\,78°\,\text{Cosec}75°}+\frac{\text{Cos}^2\,(50°+\theta)\tan^2\,(40°-\theta)}{\tan 15°\tan 37°\tan 53°\tan 75°}$$

67. Prove that :

$$\frac{1}{\text{Cosec}\,\theta+\text{Cot}\,\theta}-\frac{1}{\text{Sin}\,\theta}=\frac{1}{\text{Sin}\,\theta}-\frac{1}{\text{Cosec}\,\theta-\text{Cot}\,\theta}$$

68. If $\frac{\text{Cos}\,\alpha}{\text{Cos}\,\beta}=m$ and $\frac{\text{Cos}\,\alpha}{\text{Sin}\,\beta}=n$, then prove that $(m^2+n^2)\,\text{Cos}^2\,\beta=n^2$

69. If $\tan\theta+\text{Sin}\,\theta=m$. $\tan\theta-\sin\theta=n$, then prove that $m^2-n^2=4\sqrt{mn}$

70. Prove that :

$$\text{Sec}^2\,\theta-\frac{\text{Sin}^2\,\theta-2\text{Sin}^4\,\theta}{2\text{Cos}^4\,\theta-\text{Cos}^2\,\theta}=1$$

71. $\text{Cot}\,\theta\tan(90°-\theta)-\text{Sec}\,(90°-\theta)\,\text{Cosec}\,\theta+\sqrt{3}\tan 12°\tan 60°\tan 78°$ find its value.

72. Find the value of—

$$\frac{\text{Sec}\,(90°-\theta)\,\text{Cosec}\,\theta-\tan(90°-\theta)\,\text{Cot}\,\theta+\text{Cos}^2\,25°+\text{Cos}^2\,65°}{3\tan 27°\tan 63°}$$

ANSWERS AND HINTS

1. 45°

2. 30°

3. 24°

4. $\frac{625}{168}$

5. 7 **6.** $\frac{1}{3}$

7. a **8.** 0

9. $\frac{1}{5}$ **10.** 9

11. $\frac{\sqrt{1+\text{Cos}^2\,\theta}}{\text{Cot}\,\theta}$ **12.** 0°

13. 50° **14.** $\tan^2\theta$

15. $\tan\theta$ **16.** $\frac{1}{2}$

17. 2 **18.** 0

19. – 6 **20.** sec 42° + cot 2°

21. (c) **22.** (d)

23. (a) **24.** (a)

25. (a) **26.** —

27. — **28.** —

29. — **30.** A = 45°, B = 15°

31. 1 **32.** —

33. 22° **34.** $\frac{17}{8}$

35. 20° **36.** Hint : A + B + C = 180°

37. AC = 10 cm, BC = $5\sqrt{3}$ cm **38.** 30°

40. 60° **49.** $\sqrt{3}$

51. – 1 **53.** 2

56. $\frac{20+9\sqrt{3}}{4+3\sqrt{3}}$ **60.** $\frac{2+2\sqrt{3}}{2}$

71. 2 **72.** $\frac{2}{3}$

PRACTICE-TEST

Introduction to Trigonometry

Time : 1 Hrs. ***M.M.: 20***

SECTION-A

1. If $\text{Sin}\,\theta = \frac{4}{5}$ what is the value of $\cos\theta$. **1**

2. Write the value of $\text{Sin}\,(45° + \theta) - \text{Cos}\,(45° - \theta)$. **1**

3. If $\cos 9\alpha = \sin\alpha$ and $9\alpha < 90°$, then the value of $\tan 5\alpha$ is **1**

(a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) 1 (d) 0

4. If $\sin A + \sin^2 A = 1$, then the value of $(\cos^2 A + \cos^4 A)$ is **1**

(a) 1 (b) $\frac{1}{2}$ (c) 2 (d) 3

SECTION-B

5. If $5\tan\theta = 4$ then find the value of $\frac{5\,\text{Sin}\,\theta - 3\text{Cos}\,\theta}{5\,\text{Sin}\,\theta + 2\text{Cos}\,\theta}$ **2**

6. Find the value of $\tan 35° \tan 40° \tan 45° \tan 50° \tan 55°$ **2**

7. Prove that $(\sin\alpha + \cos\alpha)(\tan\alpha + \cot\alpha) = \sec\alpha + \text{cosec}\,\alpha$ **2**

SECTION-C

8. Prove that $\frac{\text{Sin}\,\theta}{1 + \text{Cos}\,\theta} + \frac{1 + \text{Cos}\,\theta}{\text{Sin}\,\theta} = 2\,\text{Cosec}\,\theta$ **3**

9. Prove that $\frac{\text{Cos}\,A}{1 - \tan A} - \frac{\text{Sin}^2 A}{\text{Cos}\,A - \text{Sin}\,A} = \text{Sin}\,A + \text{Cos}\,A$ **3**

SECTION-D

10. Prove that $\frac{\tan\theta + \text{Sec}\,\theta - 1}{\tan\theta - \text{Sec}\,\theta + 1} = \frac{\text{Cos}\,\theta}{1 - \text{Sin}\,\theta}$. **4**

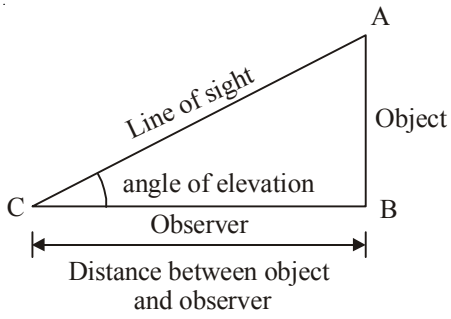
□□□

CHAPTER

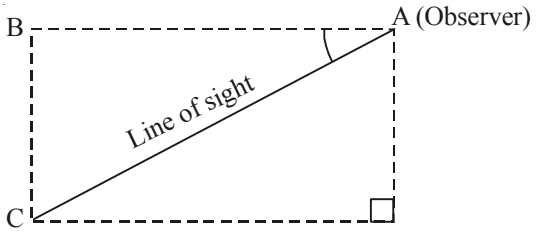
9 Some Applications of Trigonometry

KEY POINTS

- Applications of trigonometry involve finding heights of the objects and distance between them. Without actual measurement.
- **Angle of Elevation:** Let AB be an object standing vertically on a plane CB. C is the observer looking upto to A (the top of AB). AC is called the line of sight and $\angle ACB$ is angle is elevations.



- **Angle of Depression :** Let A is the observer looking at C (the object) from a height BC. AC is line of sight and $\angle BAC$ is angle of depression.



- If the observer moves towards the object the angle of elevation increases and if the observer moves away from the object, the angle of depression decreases.
- Numerically, angle of elevation is equal to angle of depression (both are measured) with the same horizontal parallel planes).

VERY SHORT ANSWER TYPE QUESTIONS

1. The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. The angle of elevation of sun is : (CBSE 2017)

(a) 45° (b) 30° (c) 60° (d) 90°

2. The tops of the poles of height 16 m and 10 m are connected by a wire of length l metres. If the wire makes an nagle of 30° with the horizontal, the l =

(*a*) 26 m (b) 16 m (c) 12 cm (d) 10 m

3. A pole of height 6 m casts a shadow $2\sqrt{3}$ m long on the ground. the angle of elevation of the sun is (CBSE 2017)

(a) 30° (b) 60° (c) 45° (d) 90°

4. A ladder leaning aginast a wall makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, then the length of the ladder is — (CBSE 2016)

(a) 3 m (b) 4 m (c) 5 m (d) 6 m

5. If a tower is 30 m hight, costs a shadow $10\sqrt{3}$ m long on the ground, then the angle of elevation of the sun is: (CBSE, 2017)

(a) 30° (b) 45° (c) 60° (d) 90°

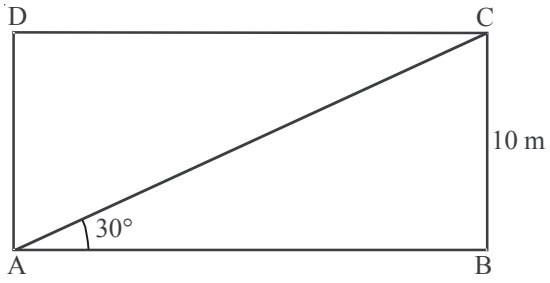
6. A tower is 50 m high. When the sun's altitude is 45° then what will be the length of its shadow?

7. The length of shadow of a pole 50 m high is $\frac{50}{\sqrt{3}}$ m. find the sun's altitude.

8. Find the angle of elevation of a point which is at a distance of 30 m from the base of a tower $10\sqrt{3}$ m high.

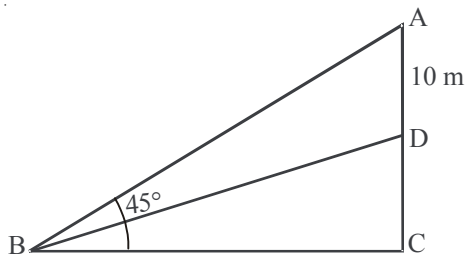
9. A kite is flying at a height of $50\sqrt{3}$ m from the horizontal. It is attached with a string and makes an angle 60° with the horizontal. Find the length of the string.

10. In the given figure find the perimeter of rectangle ABCD.



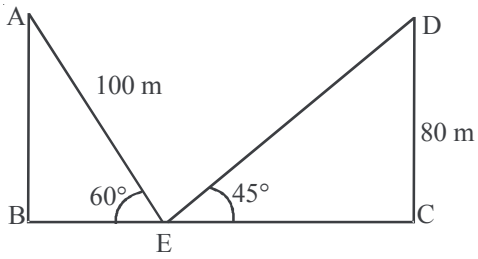
11. The length of the shadow of a pillar is $\sqrt{3}$ times its height. Find the angle of elevation of the source of light.

12. In the figure, find the value of DC.

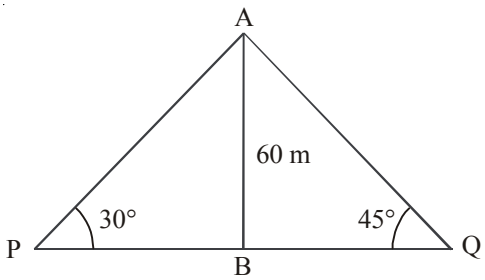


SHORT ANSWER TYPE QUESTIONS

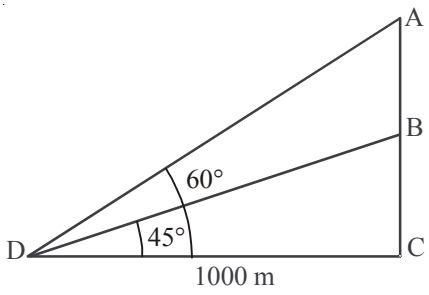
13. In the figure, find the value of BC.



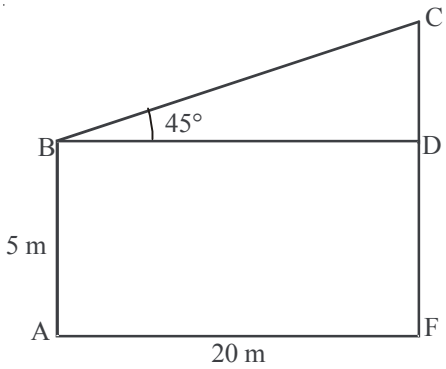
14. In the figure, two persons are standing at the opposite direction P & Q of the tower. If the height of the tower is 60 m then find the distance between the two persons.



15. In the figure, find the value of AB.



16. In the figure, find the value of CF.



17. If the horizontal distance of the boat from the bridge is 25 m and the height of the bridge is 25 m, then find the angle of depression of the boat from the bridge.

18. State True/False.
If the length of the shadow of a tower is increasing, then the angle of elevation of the sun is also increasing.

19. If a man standing on the deck of a ship 3 m above the surface of sea observes a cloud and its reflection in the sea, then the angle of elevation of the cloud is equal to the angle of depression of its reflection.

20. The angle of elevation of the top of the tower is 30°. If the height of the tower is doubled, then the angle of elevation of its will also bed doubled.

21. From the top of a hill, the angles of depression of two consecutive stones due east are found to be at 30° and 45°. Find the height of the hill.

22. The string of a kite is 150 m long and it makes an angle 60° with the horizontal. Find the height of the kite above the ground. (Assume string to be tight)

23. The shadow of a vertical tower on level ground increases by 10 m when the altitude of the sun changes from 45° to 30°. Find the height of the tower.

24. An aeroplane at an altitude of 200 m observes angles of depression of opposite points on the two banks of the river to be 45° and 60°, find the width of the river.

25. The angle of elevation of a tower at a point is 45°. After going 40 m towards the foot of the tower, the angle of elevation of the tower becomes 60°. Find the height of the tower.

26. The upper part of a tree broken over by the wind makes an angle of 30° with the ground and the distance of the root from the point where the top touches the ground is 25 m. What was the total height of the tree?

27. A vertical flagstaff stands on a horizontal plane. From a point 100 m from its foot, the angle of elevation of its top is found to be 45°. Find the height of the flagstaff.

28. The length of a string between kite and a point on the ground is 90 m. If the string makes an angle with the level ground and $\sin\alpha = \frac{3}{5}$. . Find the height of the kite. There is no slack in the string.

29. An aeroplane, when 3000 m high, passes vertically above another plane at an instant when the angle of elevation of two aeroplanes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the two planes.

30. A 7 m long flagstaff is fixed on the top of a tower on the horizontal plane. From a point on the ground, the angle of elevation of the top and the bottom of the flagstaff are 45° and 30° respectively. Find the height of the tower.

31. From the top of a 7 m high building, the angle of elevation of the top of the tower is 60° and the angle of depression of the foot of the tower is 30°. Find the height of the tower.

32. Anand is watching a circus artist climbing a 20m long rope which is tightly stretched and tied from the top of vertical pole to the ground. Find the height of the pole if the angle made by the rope with the ground level is 30°.

LONG ANSWER TYPE QUESTIONS

33. The angle of elevation of a cloud from a point 60 metres above a lake is 30° and the angle of depression of its reflection of the cloud in the lake is 60°. Find the height of the cloud.

34. A man standing on the deck of a ship, 10 m above the water level observes the angle of elevation of the top of a hill as 60° and angle of depression the bottom of a hill as 30°. Find the distance of the hill from the ship and height of the hill.

35. From a window 60 m high above the ground of a house in a street, the angle of elevation and depression of the top and the foot of another house on the opposite side of the street are 60° and 45° respectively. Show that the height of opposite

house is $60(1+\sqrt{3})$ metres.

36. The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 30 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of $3600\sqrt{3}$ m, find the speed in km/hour of the plane.

37. A bird is sitting on the top of a tree, which is 80 m high. The angle of elevation of the bird, from a point on the ground is 45°. The bird flies away from the point of observation horizontally and remains at a constant height. After 2 seconds, the angle of elevation of the bird from the point of observation becomes 30°. Find the speed of flying of the bird.

38. The angles of elevation of the top of a tower from two points on the ground at distances 9 m and 4 m from the base of the tower are in the same straight line with it are complementary. Find the height of the tower.

39. A boy standing on a horizontal plane finds a bird flying at a distance of 100 m from him at an elevation of 30°. A girl, standing on the roof of 20 m high building, finds the angle of elevation of the same bird to be 45°. Both the boy and girl are on the opposite sides of the bird. Find the distance of bird from the girl.

40. An observer from the top of a light house, 100 m high above sea level, observes the angle of depression of a ship, sailing directly towards him, changes from 30° to 60°. Determine the distance travelled by the ship during the period of observation.

41. The angles of elevation and depression of the top and bottom of a light house from the top of a building 60 m high are 30° and 60° respectively. Find

(i) The difference between the height of the light house and the building.

(ii) distance between the light house and the building.

42. A fire in a building 'B' is reported on telephone in two fire stations P an Q, 20 km apart from each other on a straight road. P observes that the fire is at an, angle of 60° to the road, and Q observes, that it is at an angle of 45° to the road. Which station should send its team to start the work at the earliest and how much distance will this team has to travel?

43. A 1.2m tall girl spots a balloon on the eve of Independence Day, moving with the wind in a horizontal live at a height of 88.2 m from the ground. The angle of elevation of the balloon from the of the girl at an instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon.

44. The angle of elevation of the cloud from a point 60 m above take is 30° and the angle of depression of the reflection of the cloud in the take is 60°. Find the height of the cloud. (CBSE, 2011 C)

45. The pillars of equal heights stand on either side of a roadway 150 m wide From a joinj on the roadway between the pillars, the angles of elevation of the top of the pillars are 60° and 30°. Find the height of pillars and the position of the point. (CBSE, 2011)

46. The angle of elevation of the top of tower from certain point is 30°. If the observer moves 20 m towards the tower the angle of elevation of the top increases by 15°. Find the height of the tower.

47. A moving boat is observed from the top of a 150 m high diff moving away form the cliff. The angle of depression of the boat changes form 60° to 45° in 2 minutes. Find the speed of the boat in m/w.

(CBSE 2017)

48. From the top of a 120 m hight tower a man observes two cars on the opposite sides of the tower and in straight line with the base of tower with angles of depression as 60° and 45°. Find the distance between the cars. (CBSE, 2017)

49. From the top of a tower h metre high, the angles of depression of two objects, which are in the line with the foot of the tower are α & β ($\beta > \alpha$). Find the distance between the two objects. (NCERT, Exampler)

50. A window of a house is h metres above the ground. From the window the angles of elevation and depression of the top and bottom of another house situated on the opposite side of the lane are found to be α & β respectively. Prove that the height of the house is $h\,(1 + \tan\alpha \times \tan\beta)$ metres.

(NCERT Exampler)

ANSWERS AND HINTS

1. (b) **2.** (c)

3. (b) **4.** (c)

5. (c) **6.** 50 m

7. 60° **8.** 30°

9. 100 m

10. $20\left(\sqrt{3}+1\right)$ m

11. 30°

12. 60 m

13. 130 m

14. $60\left(\sqrt{3}+1\right)$ m

15. $1000\left(\sqrt{3}-1\right)$ m

16. 25 m

17. 45

18. False

19. False

20. False

21. 1.37 km.

22. $75\sqrt{3}$ m

23. 13.65 m

24. 315.8 m

25. 94.8 m

26. 43.3 m

27. 100 m

28. 120 m

29. 1268 m

30. 9.6 m

31. 28 m

32. 10 m

33. 120 m

34. 40 m, 17.32 m

36. 864 km/hour

37. 29.28 m

38. 6 m

39. $30\sqrt{2}$ m

40. 115.5 m

41. 20 m, 34.64 m

42. Station P, 14.64 km

43. $58\sqrt{3}m$

44. 120 m

45. height = 64.95 m, distance (Position) = 112.5 m

46. $10(\sqrt{3}+1)\ m$

47. 1902 m/h

48. 189.28 m

49. $h(\cot\alpha - \cot\beta)$ m

PRACTICE-TEST

Heights and Distances

Time : 1 Hr. ***M.M.: 20***

SECTION-A

1. A pole which is 6 m high cast a shadow $2\sqrt{3}$ on the ground. What is the sun's angle of elevation. 1
2. The height of a tower is 100 m. When the angle of elevation of sun is 30°, then what is the shadow of tower? 1
3. The angle of elevation of the sun, when the shadow of a pole h meters high is $\sqrt{3}\,h$ is.

   (*a*) 30° (*b*) 45° (*c*) 60° (*d*) 90°
4. An observer 1.5 metre tall is 20.5 metre away from a tower 22 metres high. The angle of elevation of the top of the tower from the eye of the observer is,

   (*a*) 30° (*b*) 45° (*c*) 60° (*d*) 0° **1**

SECTION-B

5. From a point on the ground 20 m away from the foot of a tower the angle of elevation is 60°. What is the height of tower? 2
6. The ratio of height and shadow of a tower is $1:\frac{1}{\sqrt{3}}$ .What is the angle of elevation of the sun? 2
7. The angle of elevation of the top of a tower is 30°. If the height of the tower is tripled, then prone that the angle of elevation would be doubled. 2

SECTION-C

8. The tops of the two towers of heigth x and y standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find $x : y$. 3
9. The angle of elevation of the top of a rock from the top and foot of a 100 m high tower are 30° and 45° respectively. Find the height of the rock. 3

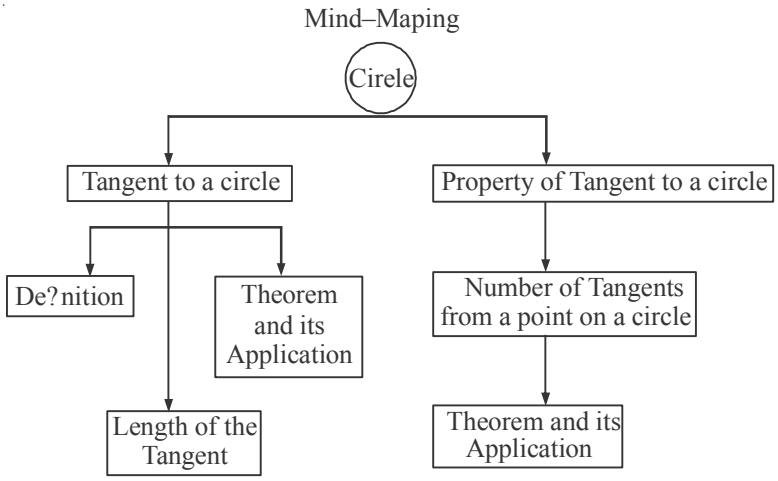
SECTION-D

10 A man standing on the deck of a ship, 10 m above the water level observes the angle of elevation of the top of a hill as 60° and angle of depression of the base of the hill as 30°. Find the distance of the hill from the ship and height of the hill. 4

□□□

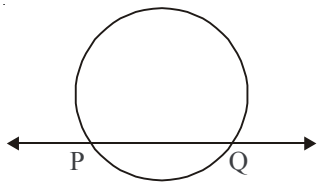
CHAPTER

10 Circles



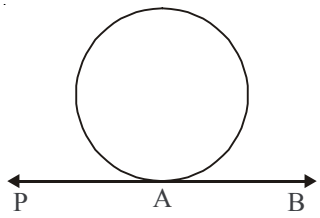
KEY POINTS

1. A **circle** is a collection of all those points in a plane which are at a constant distance from a fixed point. The fixed point is called the **centre** and fixed distance is called the **radius**.
2. **Secant:** A line which intesects a circle in two distinct points is called a secant of the circle.



3. **Tangent**: It is a line that intersects the circle at only one point. The point where tangent touches the circle is called the point of contact.

   Here A is the poin of contact.



4. **Number of Tangent**: Infinitely many tangents can be drawn on a circle.

5. **Number of Secant**: There are infinitely many secants which can be drawn on a circle.

6. The proofs of the following theorems can be asked in the examination:–

(i) The tangent at any point of a circle is perpendicular to the radius through the point of contact.

(ii) The lengths of tangents drawn from an external point to a circle are equal.

7. There is only one tangent at a point of the circle.

8. The tangent to a circle is a special case of the secant.

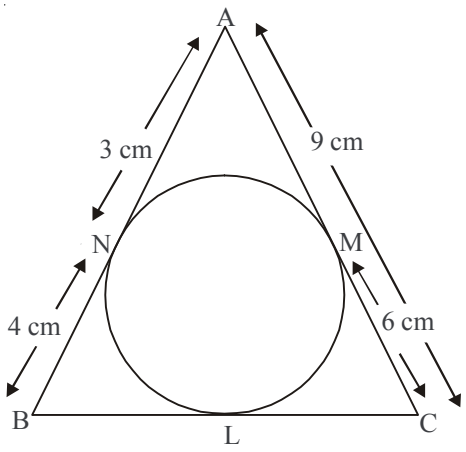
9. There is no tangent to a circle passing through a point lying inside the circle.

10. There is one and only one tangent to a circle passing through a point lying on the circle.

11. There are exactly two tangents to a circle through a point lying outside the circles.

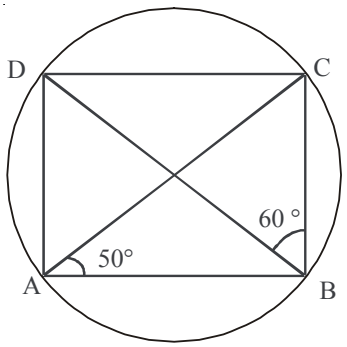
VERY SHORT ANSWER TYPE QUESTIONS

1. In fig., ΔABC is circumscribing a circle. Find the length of BC.

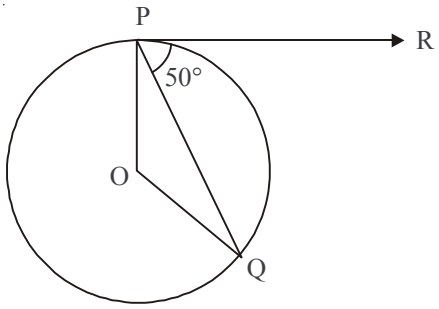


2. The length of the tangent to a circle from a point P, which is 25 cm away from the centre, is 24 cm. What is the radius of the circle.

3. In fig., ABCD is a cyclic quadrilatreral. If $\angle BAC = 50°$ and $\angle DBC = 60°$, then find $\angle BCD$.



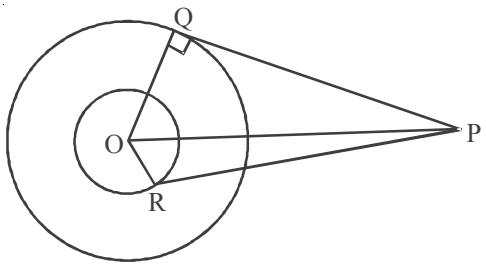
4. In figure, O is the centre of a circle, PQ is a chord and the tangent PR at P makes an angles of 50° with PQ. Find $\angle POQ$.



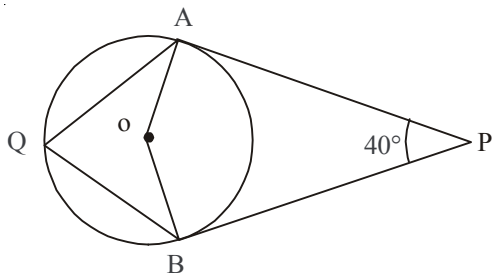
5. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

6. If radii of two concentric circles are 4 cm and 5 cm, then find the length of the chord of one circle which is tangent to the other circle.

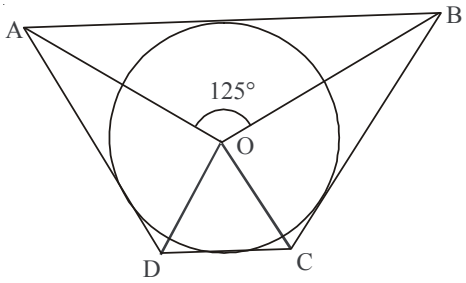
7. In the given figure, PQ is tangent to outer circle and PR is tangent to inner circle. If PQ = 4cm, OQ = 3 cm and QR = 2 cm then find the length of PR.



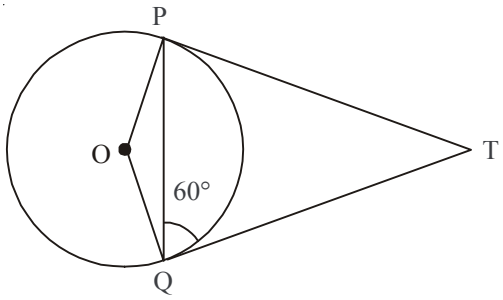
8. In the given figure, O is the centre of the circle, PA and PB are tangents to the circle then find ∠AQB. (CBSE 2016)



9. In the given figure, If ∠AOB = 125° then find ∠COD.



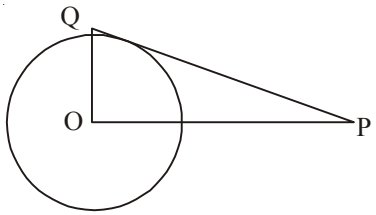
10. If two tangent TP and TQ are drawn from an external point T such that ∠TQP = 60° then find ∠OPQ.



11. How many tangents can a circle have? (NCERT)

12. A tangent to a circle intersects it in _____ points. (NCERT)

13.



If PQ is a tangent then find the value of $\overline{POQ} + \overline{QPO}$.

14. Choose the correct Answer.

A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is :

(a) 12 cm (b) 13 cm (c) 8.5 cm (d) $\sqrt{119}$ cm (NCERT)

15. A circle can have _____ parallel tangents at the most. (NCERT)

16. The common point of a tangent to a circle and the circle is called _____ . (NCERT)

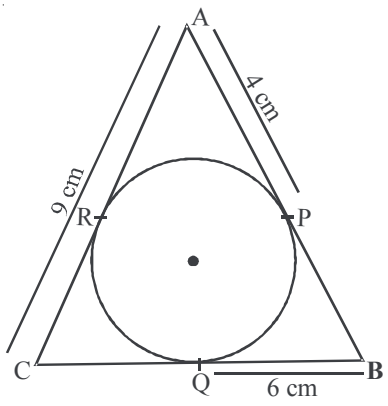
SHORT ANSWER TYPE-I QUESTIONS

17. If diameters of two concentric circle are d_1 and d_2 ($d_2 > d_1$) and c is the length of chord of bigger circle which is tangent to the smaller circle. Show that $d_2^2 = c^2 + d_1^2$.

18. The length of tangent to a circle of radius 2.5 cm from an external point P is 6 cm. Find the distance of P from the nearest point of the circle.

19. TP and TQ are the tangents from the external point T of a circle with centre O. If $\angle OPQ = 30°$ then find the measure of $\angle TQP$.

20. In the given fig. AP = 4 cm, BQ = 6 cm and AC = 9 cm. Find the semi perimeter of ΔABC.

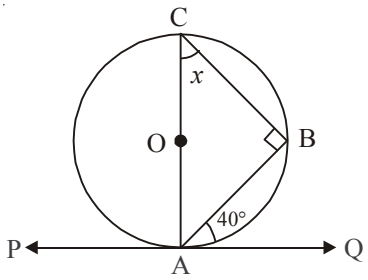


21. A circle is drawn inside a right angle triangle whose sides are a, b, c where c is the hypotenuse, which touches all the sides of the triangle. Prove $r = \dfrac{a + b - c}{2}$ where r is the radius of the circle. (NCERT Exampler, 2012)

22. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

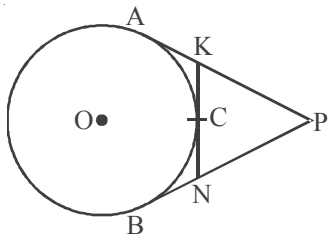
23. Prove that in two concentric circles the chord of the larger circle which is tangent to the smaller circle is bisected at the point of contact.

24. In the given Fig., AC is diameter of the circle with centre O and A is point of contact, then find x.

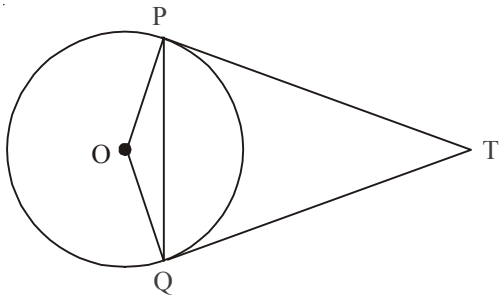


25. In the given fig. KN, PA and PB are tangents to the circle. Prove that:

KN = AK + BN.

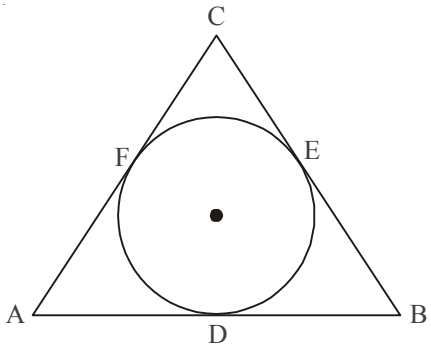


26. In the given fig. PQ is a chord of length 6 cm and the radius of the circle is 6 cm. TP and TQ are two tangents drawn from an external point T. Find ∠PTQ.

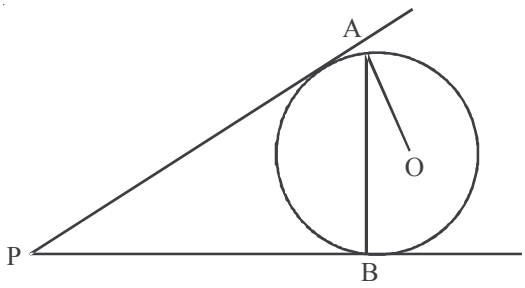


SHORT ANSWER TYPE-II QUESTIONS

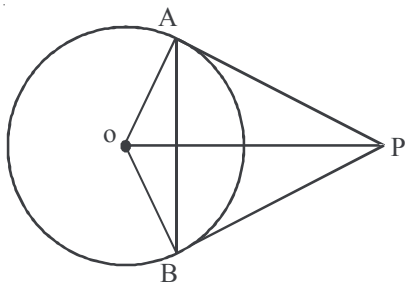
27. In the given figure find AD, BE, CF where AB = 12 cm, BC = 8 cm and AC = 10 cm.



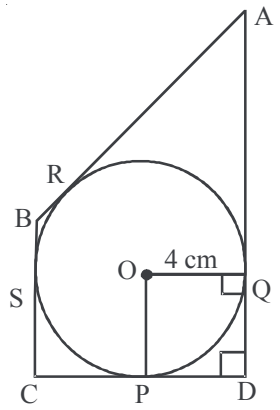
28. Two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that $\angle APB = 2\angle OAB$ (NCERT, Exemplar-2)



29. In the given fig. OP is equal to the diameter of the circle with centre O. Prove that ΔABP is an equilateral triangle.



30. In the given fig., find PC. If AB = 13 cm, BC = 7 cm and AD = 15 cm.

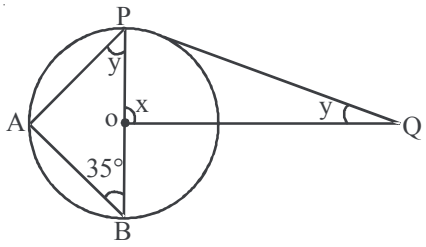


LONG ANSWER TYPE QUESTIONS

31. In the given fig. find the radius of the circle.

m

32. In the given fig. PQ is tangent and PB is diameter. Find the value of x and y.



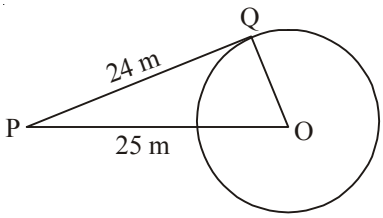
ANSWERS AND HINTS

1. Since length of both the tangents from a point outside the circle is equal, So

$$BN = BL,\ CM = CL$$

$$BL + CL = BC = 10 \text{ cm}$$

2.



By Pythagorous Rule, QR = 7 cm.

3. Angle is the same segment are euqal.

- DC is the chord so $\angle DAC = 60°$.
- The sum of the opposite angles of a cyclic quadrilateral is 180°.

So $\angle BCD = 70°$

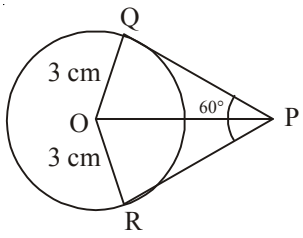
4. The tangent at any point of a circle is perpendicular to the radius through the point of contact.

So, $\angle RPO = 90°$

$\angle OPQ = \angle OQP = 40°$

$\angle POQ = 100°$

5.



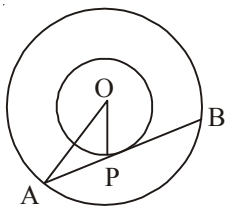
$\Delta QPO \cong \Delta RPO$

$\Rightarrow \quad \angle QPO = \angle RPO = \frac{60°}{2} = 30°$

In ΔQPO, $\angle OQP = 90°$ (Tangent is perpendicular at the point of contact).

$$\tan 30° = \frac{OQ}{QP} \Rightarrow QP = 3\sqrt{3} \text{ cm}$$

6.



In ΔAOP, right angled at P.

$$OA^2 = AP^2 + OP^2 \Rightarrow (5)^2 = AP^2 + 4^2 \Rightarrow AP^2 = 9$$

$\Rightarrow \quad AP = 3$

In ΔPQO, $AB = 6$ cm

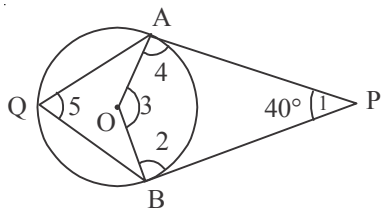
7. $(4)^2 + (3)^2 = (OP)^2$

$5 = OP$

In ΔPRO, $(5)^2 = (2)^2 + (PR)^2$

$PR = \sqrt{21}$ cm

8.



In Quadrilateral PROQ

$\underline{|1} + \underline{|2} + \underline{|3} + \underline{|4} = 360°$

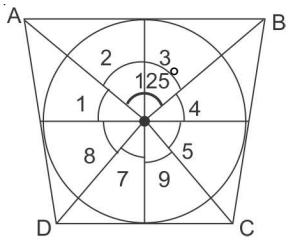
$\underline{|1} + \underline{|3} = 180°$

$\underline{|3} = 140°$

Now, $\underline{|3} = 2\underline{|5}$

$\underline{|5} = 70°$ or $\underline{|AQB} = 70°$

9. $\left.\begin{array}{l}\underline{|1} = \underline{|2} \\ \underline{|3} = \underline{|4} \\ \underline{|5} = \underline{|6} \\ \underline{|7} = \underline{|8}\end{array}\right]$ (CPCT) of their corresponding triangles.



$2(\underline{|2} + \underline{|3} + \underline{|6} + \underline{|7}) = 360°$

or $\underline{|AOB} + \underline{|COD} = 186°$

or $\underline{|COD} = 55°$

10. $\underline{|OQT} = 90°$ (Angle between tangent & radius)

$\underline{|PQO} = 30°$ $(90° - 60°)$

$\underline{|PQO} = \underline{|OPQ} = 30°$

11. Infinity many

12. One

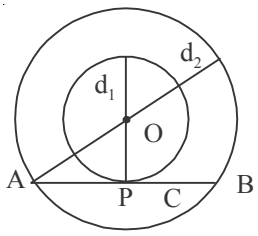
13. 90° as $\underline{|OQP} = 90°$ (Angle between tangent and radius of the circle)

14. D($\sqrt{119}$ cm)

15. Two

16. Point of Contact

17.



$$AO^2 = OP^2 + AP^2$$

$$\left(\frac{d_2}{2}\right)^2 = \left(\frac{d_1}{2}\right)^2 + AP^2$$

$$\left(\frac{d_2}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2 = AP^2$$

$$\sqrt{\frac{1}{4}\left[(d_2)^2 - (d_1)^2\right]} = AP$$

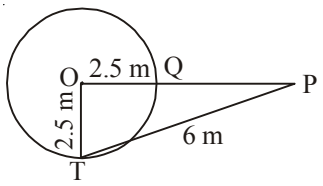
$$2\sqrt{\frac{1}{4}\left[(d_2)^2 - (d_1)^2\right]} = AB$$

$$\sqrt{(d_2)^2 - (d_1)^2} = C$$

$$(d_2)^2 - (d_1)^2 = C^2$$

$$d_2{}^2 = C^2 - d_1{}^2$$

18.



$$(OP)^2 = (OT)^2 + (PT)^2$$

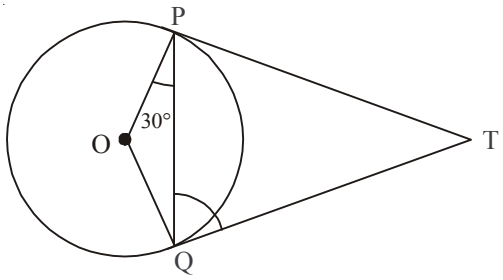
$$(OP)^2 = (2.5)^2 + (6)^2$$

$$= 42.25$$

$$(OP)^2 = (6.5)^2$$

$$QP = 4 \text{ cm}$$

19.



$\angle OQP = \angle OPQ = 30°$

$\angle OQT = 90°$ (Angle between radius tangent)

$\angle TQP = \angle OQT - \angle OQP$

$= 90° - 30° = 60°$

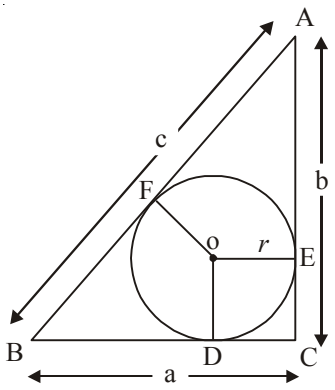
20. $AP = AR = 4$ cm

$CR = CQ = (9 - 4)$ cm $= 5$ cm

$= \frac{1}{2}[AC + AB + BC]$

$= \frac{1}{2}[9 + 10 + 11] = 15$ cm

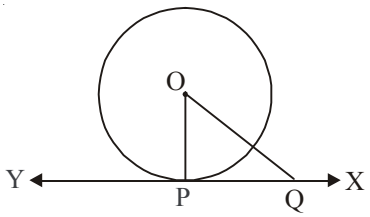
21.



$b - r = AF, \quad a - r = BF$

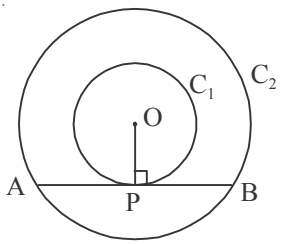
or, $AB = C = AF + BF = b - r + a - r$

This gives, $r = \frac{a + b - c}{2}$

22. 

(Theorem 10.1, NCERT)

23. Join OP



AB is tangent to C_1 at P and OP is radius

$$OP \perp AB$$

AB is chord of circle C_2 and $OP \perp AB$.

Therefore OP is the bisector of the chord AB as the perpendicular from the centre bisects the chord i.e,

$$AP = BP$$

24. $$\angle OAB = 50°$$

$$x + \underline{|B} + \underline{|OAB} = 180°$$

$$x + 90° + 50° = 180°$$

$$x = 40°$$

25. $$AK = KC \quad \ldots(1)$$

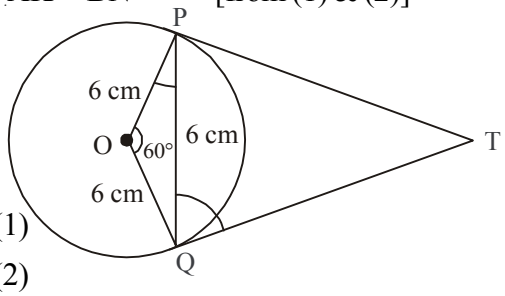
$$BN = NC \quad \ldots(2)$$

$$KN = KC + NC = AK + BN \quad [\text{from } (1) \& (2)]$$

26. $$\underline{|POQ} + \underline{|PTQ} = 180°$$

$$60° + \underline{|PTQ} = 180°$$

$$\underline{|PTQ} = 120°$$



27. $$AC = AF + FC = 10\text{ cm} \quad \ldots(1)$$

$$AB = AD + DB = 12\text{ cm} \quad \ldots(2)$$

$$BC = BE + CE = 8\text{ cm} \quad \ldots(3)$$

$$\begin{bmatrix} BD = BE \\ AD = AF \\ CF = CE \end{bmatrix} \quad \ldots(4)$$

2AD, 2FC, 2BD are obtained

Replace from (4) in (1), (2), (3) (So that in (5) + (6) + (7)). 2AD, 2FC, 2BD are obtained.

$AC = AD + FC = 10\,cm$ …(1)

$AB = AD + DB = 12\,cm$ …(2)

$BC = BD + CE = 8\,cm$ …(3)

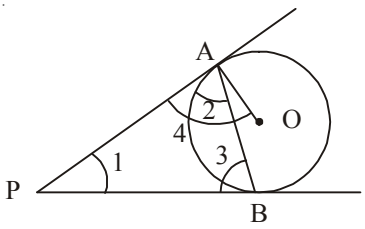
Add (5, 6, 7)

$2(AD + FC + DB) = 30$

$AD + FC + DB = 15$

Substitute values from (1), (2) & (3)
and find. AD = 7 cm, BE = 5 cm, CF = 3 cm.

28. $PA = PB$



So, $\angle 2 = \angle 3 = \frac{1}{2}(180° - \angle 1)$

$\angle 2 = \angle 3 = 90° - \frac{1}{2}\angle 1$

$\angle 4 = 90°$ (Angle between tangent & Radius)

$\angle OAB = \angle 4 - \angle 2$

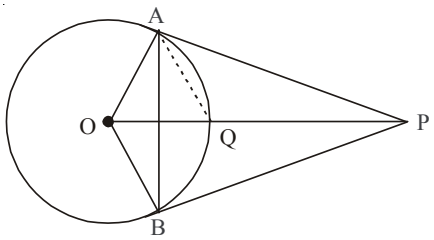
$= 90° - \left(90° - \frac{1}{2}\angle 1\right) = 90° - 90° + \frac{1}{2}\angle 1$

$\angle OAB = \frac{1}{2}\angle APB$

$2\angle OAB = \angle APB$

29. $OP = 2r$

$\Rightarrow$ $QP = QP = r$



Consider ΔAOP is which $OA \perp AP$ and OP is the hypotenuse.

$OQ = AQ = OA$

(Mid point of hy potenuse is equidistance from the vertices).

$\Rightarrow$ OAQ is an equilitateral triangle.

$\Rightarrow$ $\angle AOQ = 60°$

Consider right angled triangle OAP

$\angle AOQ = 60°$

$$\angle OAP = 90° \Rightarrow \angle APO = 30°$$

$$\angle APB = 2\angle APO = 2 \times 30° = 60°$$

$$PA = PB \text{ (tangents)}$$

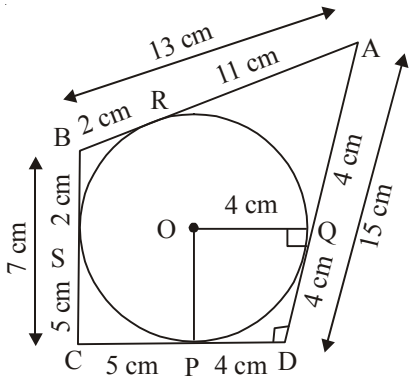
$$\Rightarrow \quad \angle PAB = \angle PBA$$

$$\text{In } \Delta APB = 60°$$

$$\angle PAB = \angle PBA = \frac{180° - 60°}{2} = 60°$$

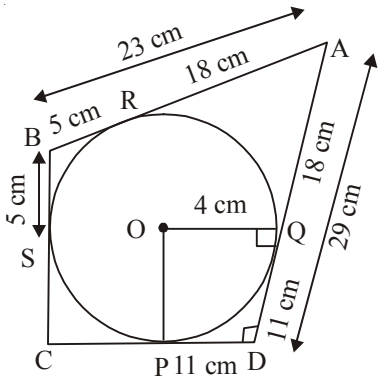
$\Rightarrow$ each angle of DPAB = 60°. **Hence Proved.**

30.



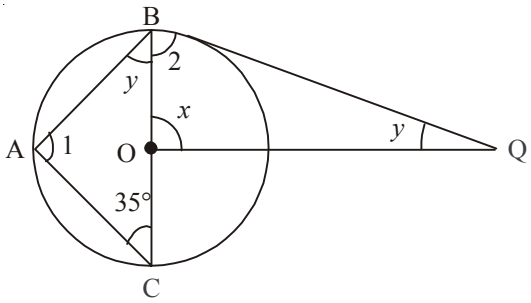
PC or CP = 5 cm

31.



r = 11 cm

32.



In ΔABC, $\angle 1 = 90°$ (Angle in semi-circles)

$\angle 1 + 35° + y = 180°$

$90° + 35° + y = 180°$

$y = 55°$

ΔOBQ, $\angle 2 = 90°$ (Angle between tangent and radius)

$\angle 2 + \angle x + \angle y = 180°$

$90° + \angle x + 55° = 180°$

$x = 35°$

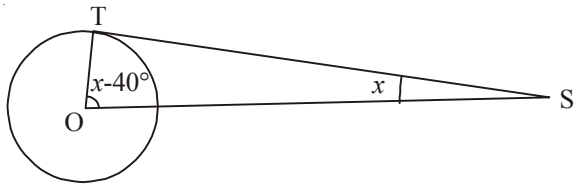
PRACTICE-TEST

CIRCLES

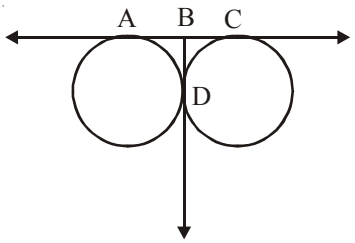
Time : 1 Hr. ***M.M.: 20***

SECTION-A

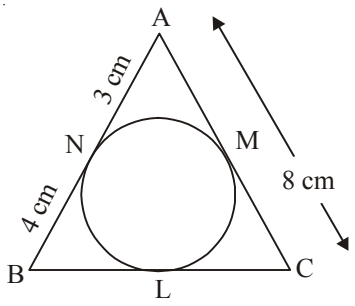
1. In the given figure find x, where ST is the tangent. 1



2. In the given figure if AC = 9, find BD. 1



3. In the given figure, ΔABC is circumscribing a circle, then find the length of BC. 1

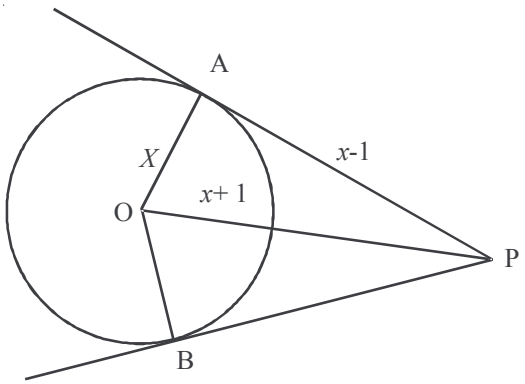


4. From the external point P tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, then find ∠AOB. (Delhi-2016, CBSE) 1

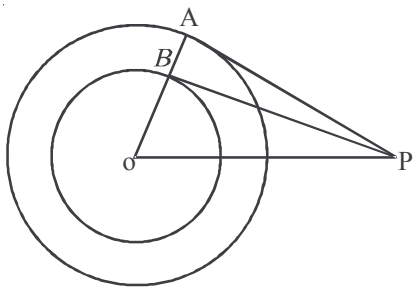
SECTION-B

5. If the angle between two tangents drawn from an external point P to a circle of radius a and centre O is 60° then find the length of OP. (All India 2017) 2

6. In the following figure find x. 2

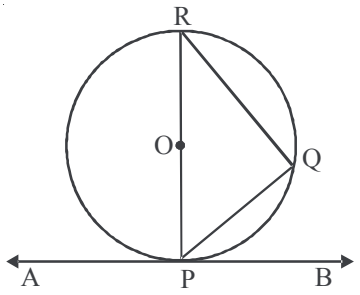


7. Two concentric circle with centre O are of radii 6 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circle as shown in the figure. If AP = 10 cm. Find BP 2



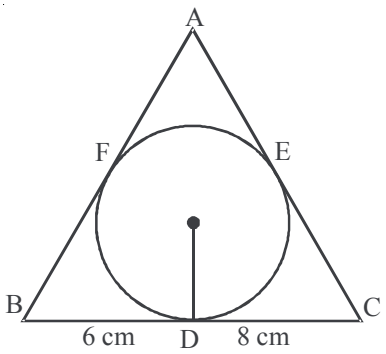
SECTION-C

8. In the given figure, AB is a tangent to a circle with centre O. Prove $\angle BPQ = \angle PRQ$. 3



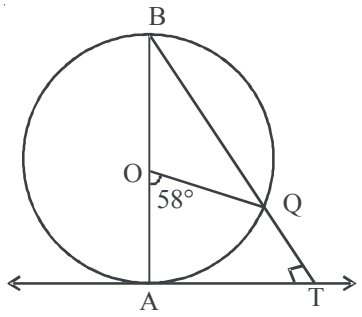
9. In the given figure ΔABC is drawn to circumscribe a circle of radius 3 cm, such that the segment BD and DC into which BC is divided by the point of

contact D are of length 6 cm and 8 cm respectively, find side AB if the *ar*(ΔABC) = 63 cm² 3



SECTION-D

10. AB is a diameter of a circle with centre O and AT is a tangent. If ∠AOQ = 58° find ∠ATQ. 4



❐❐❐

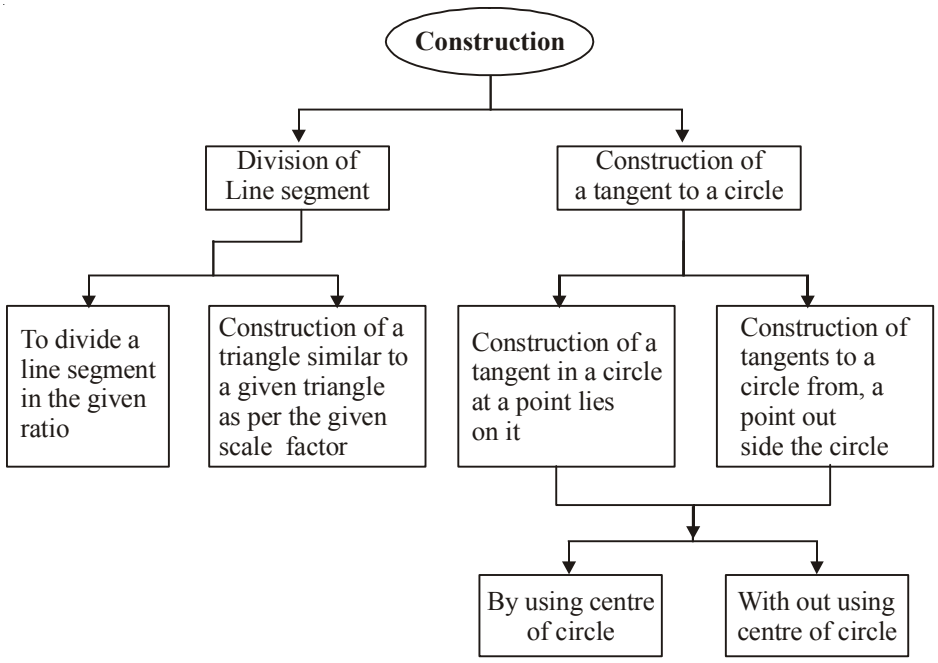
CHAPTER

11 Constructions

TOPICS

- Division of a line segment.
- Construction of a Triangle.
- Construction of Tangents of a Circle.

MIND MAPING



KEY POINTS

1. Construction should be neat and clean and there should be no donbling.
2. Construction should be as per a given scale factor which may be less than 1 or greater than 1 for a triangle similar to a given triangle.
3. Step of construction should be provided only when it is mentioned in the question.

4. We make use of compass and ruler only but in case of non-standard angles, protractor can be used.
5. Divide a line segment in the given ratio means to determine a point on the given line segment which divides it in the the given ration.
6. A tangent to a circle is a straight line which touches the circle at a point. This point is called the point of content and the radius through the point of contact is perpendicular to the tangent.
7. Tangents drawn from an external point to a circle are equal.

VERY SHORT ANSWER TYPE QUESTIONS

1. Construct a triangle similar to a given ΔABC with its sides $\frac{5}{3}$ of the corresponding sides of ΔABC, a ray BX is drawn such that CBX is an acute angle and X is on the opposite side od A with respect to BC. What is the minimum no. of points to be located at equal distances on ray BX.
2. Draw a pair of tangents to a circle which are inclined to each other at an angle of 30°. What should be the angle between two radii?
3. Constract a triangle similar to a given ΔABC with its sides $\frac{2}{5}$ of the corresponding sides of ΔABC , firstly a ray BX is drawn such that CBX is an acute angle and X lies on the opposite side of A with respect to BC then points B_1, B_2, B_3, are located on BX at equal distances Which two points will be joined in the next step.
4. Divide a line segment AB in the ratio 3:7, What is the minimum number of points marked on a ray AX at equal distances?
5. How many tangents can be drawn from a point lying inside a circle?
6. Divide a line segment AB in the ratio 4:5, a ray AX is drawn first such that $\angle BAX$ is an acute angle and then points A_1, A_2, A_3, ... are located at equal distances on the ray AX which should be joined to B?
7. Divide a line segment AB in the ratio 4:5, the points A_1, A_2, A_3,....and B_1, B_2, B_3,.... are located at equal distances on the ray AX and BY respectively. Which two points should be joined to divide a line segment?
8. Draw a line segment of length 6 cm. Find a point P on it which divides it in the ratio 3 : 4. (Delhi-2011)

9. Draw a line segment AB = 8 cm and divide it internally in the ration 3 : 2.

10. Draw a line segment AB of length 6.5 cm. Find a point P on it such that $\frac{AP}{AB} = \frac{3}{5}$

11. Geometrically divide a line segment of length 8.4 cm in the ratio 5 : 2. (forign–2011)
CBSE – 2015

12. Is it possible to div ide a line segment in the ration $\sqrt{5} : \frac{1}{\sqrt{5}}$ by geometrical construction?

13. Draw a line segment of length 7.6 cm and divide it in t he ratio 3 : 2. (Foreign – 2011)

14. Write True or False.
By geometrical construction, it is possible to divide a line segment in the ratio $\sqrt{3} : \frac{1}{\sqrt{3}}$. (NCERT Exampler)

15. Is it possible to construct a pair of tangents from point P to circle of rarius 5 cm situated at a distance of 4.9 cm from the centre?

16. Is it possible to construct a pair of tangents from point A lying on the circle of radius 4 cm and centre O.

17. Compare the length of the tangents drawn from the external point to circle.

LONG ANSWER TYPE QUESTIONS

18. AB is a line segment of length 8 cm. Locate a point C on AB such that $AC = \frac{1}{3} CB$.

19. Construct a ΔABC in which AB = 6.5 cm, ∠B = 60° and BC = 5.5 cm. Also construct a triangle AB'C' similar to ΔABC, whose each side is $\frac{3}{2}$ times the corresponding sides of ΔABC.

20. Construct a ΔABC in which BC = 5 cm, CA = 6 cm and AB = 7. Construct a ΔA'BC' similar to ΔABC, each of whose side are times $\frac{7}{5}$ the corresponding sides of ΔABC.

21. Construct a triangle with side 4 cm, 5 cm, 7 cm. Then construct a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the given triangle.

22. Construct a right triangle in which sides (other than hypotenuse) are of lengths 8 cm and 6 cm. Then construct another triangle similar to this triangle whose sides are times the corresponding sides of the first triangle.

23. Construct a ΔABC in which BC = 8 cm, $\angle B = 45°$ cm and $\angle C = 30°$. Construct another triangle similar to ΔABC such that each side are $\frac{3}{4}$ of the corresponding sides of ΔABC

24. A triangle ABC is given such that AB = 4 cm, BC = 7 cm and $\angle BAC = 50°$. Draw another triangle A'BC' similar to ΔABC with sides BA' and BC' equal to 6 cm and 10.5 cm respectively. Find the scale factor.

25. Draw a pair of tangents to a circle of radius 6 cm which are inclined to each other at an angle of 60°. Also justify the construction.

26. Construct a triangle ABC in which AB = 5 cm, $\angle B = 60°$ and attitude CD = 3 cm. Construct a $\Delta AQR \sim \Delta ABC$ such that each sides is 1.5 times that of the corresponding sides of ΔABC.

27. Draw an isosceles ΔABC with AB=AC and base BC=7cm, vertical angle is 120°. Construct $\Delta AB'C' \sim \Delta ABC$ with its sides $1\frac{1}{3}$ times of the corresponding sides of ΔABC.

28. Draw a circle of radius 3 cm. From a point 5 cm from the centre of the circle, draw two tangents to the circle. Measure the length of each tangent.

29. Draw a circle of radius 4 cm with centre O. Draw a diameter POQ. Through P or Q draw a tangent to the circle.

30. Draw two circle of radius 5 cm and 3 cm with their centres 9 cm apart. From the centre of each circle, draw tangents to other circles.

31. Draw two circles of radii 6 cm and 4 cm. From a point on the outer circle, draw a tangent to the inner circle and measure its length.

32. Draw a circle of radius 3 cm. Take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points.

33. Draw a line segment PQ = 10 cm. Take a points A on PQ such that $\frac{PA}{PQ} = \frac{2}{5}$ Measure the length of PA and AQ

34. Draw an equilateral triangle PQR with side 5cm. Now construct ΔPQ′R′ ~ ΔPQR such that $\frac{PQ}{PQ'} = \frac{1}{2}$.

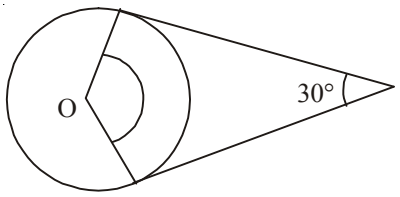
35. Draw a line segment of length 8 cm and divided it in the ratio 5:8. Meeasure the two parts.

36. Construct a triangle ABC with sides AB = 7 cm, BC = 7.5 cm and CA = 6.5 cm. Construct a Δ similar to ΔABC whose sides are $\frac{3}{2}$ of the corresponding sides of ΔABC.

ANSWERS AND HINTS

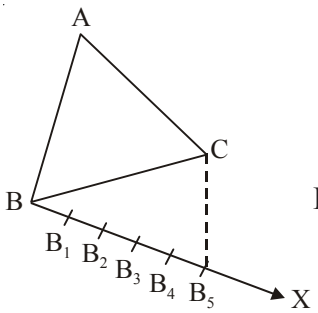
1. Since the ratio is $\frac{5}{3}$, 5 is the larger number so Answer is 5.

2.



Sum of both the angles shown in figure is 180° if one is 30° the other will be 150°.

3.



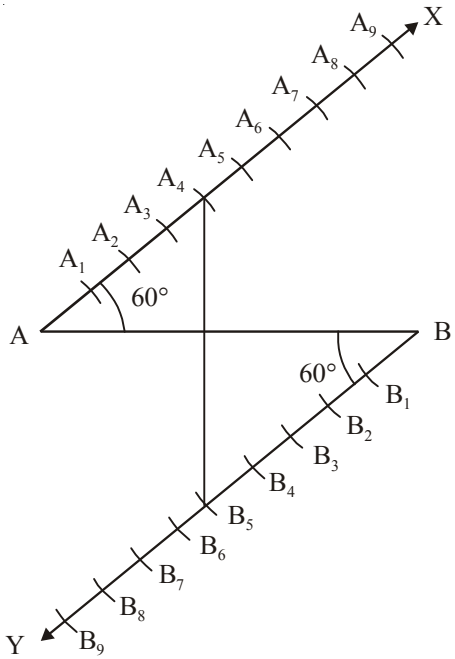
B_5 to C

4. 3 + 7 = 10

5. 0

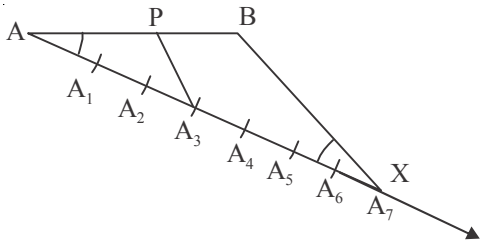
6. As shown in question (3) above it should be A_9

7.

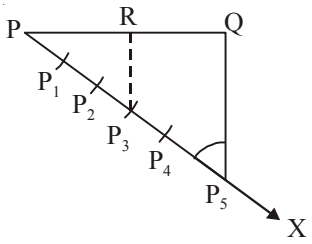


A_4 & B_5

8.



9.



10. Similar to Example 1 (NCERT)

11. As above Question-9.

12. Yes, as $\sqrt{5} : \frac{1}{\sqrt{5}} = 5 : 1$

13. As above question No. 9.

14. True as $\sqrt{3} : \frac{1}{\sqrt{3}}$ can be simplified as 3 : 1.

15. No

16. No

17. Equal.

Questions No. 18 to 36.

Questions are similar to examples given in NCERT. Please refer NCERT example.

PRACTICE-TEST

CONSTRUCTIONS

Time : 1 Hrs. ***M.M.: 20***

SECTION-A

1. Draw a perpendicular bisector of line segment AB = 8cm. 1
2. Draw a line parallel to a given line. 1
3. Draw the tangent to a circle of diameter 4 cm at a point *P* on it. 1
4. Draw two tangents to a circle of radius 4 cm from a point T at a distance of 6 cm from its centre. 1

SECTION-B

5. Draw a pair of tangents to a circle of radius 5 cm, which are inclined to each other at an angle of 60°. (Foreign - 2014) 2
6. Draw an angle bisectorof 75°. 2
7. Draw a line segment of 5.6cm. Divide it in the ratio 2:3. 2

SECTION-C

8. Draw two tangents to a circle of radius 3.5cm from a point P at a distance of 5.5cm from its centre. Measure its length. 3
9. Draw a circle of radius 3.5cm. Draw two tangents to the circle such that they include an angle of 120°. 3

SECTION-D

10. Construct a ΔABC of sides AB = 4cm, BC = 5cm and AC = 7 cm.Construct another triangle similar to ΔABC such that each of its sides is $\frac{5}{7}$ of the corresponding sides of ΔABC. 4

□□□

CHAPTER

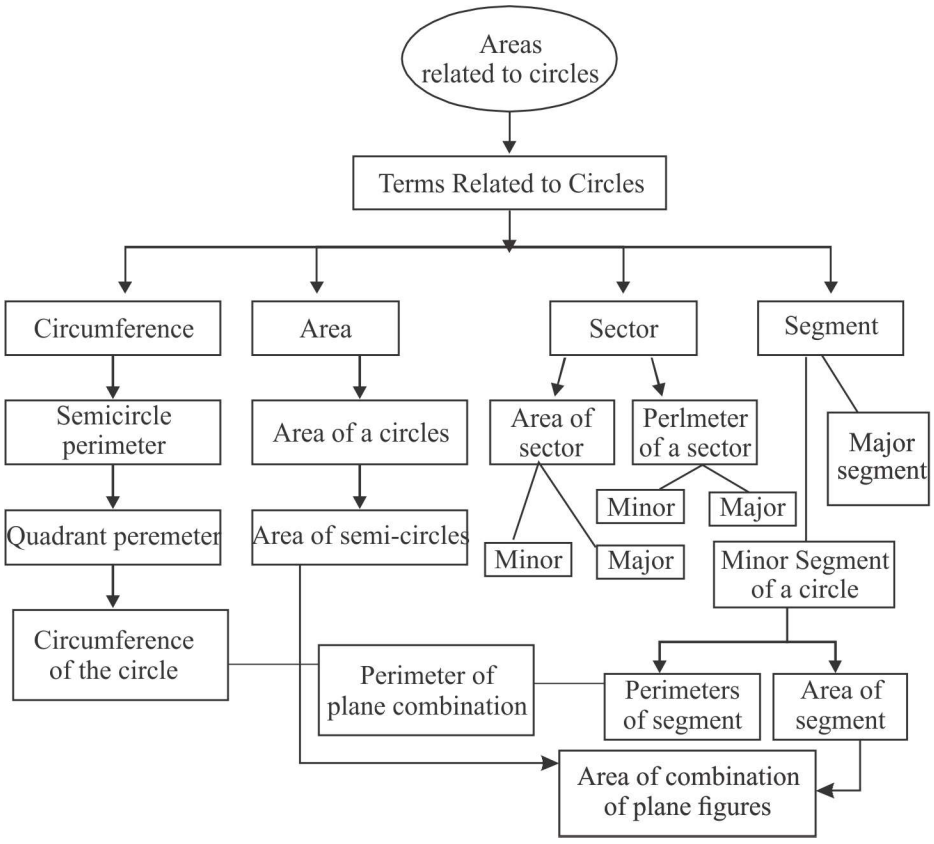
12 Areas Related to Circles

TOPICS

Perimeter and Area of a circle.

Area of sector and semgnet of a circle.

MIND MAPING



KEY POINTS

Circle: A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point always remains the same. The fixed point is called the

centre and the constant distance is known as the radius of the circle.

If r is radius of a circle, then

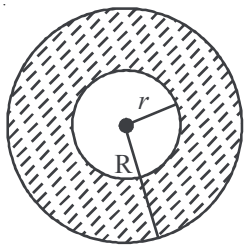
(i) Circumference = $2\pi r$ or πd where d = 2r is the diameter of the circle

(ii) Area = πr^2 or $\frac{\pi d^2}{4}$

(iii) Area of semi circle = $\frac{\pi r^2}{2}$

(iv) Area of quadrant of a circle = $\frac{\pi r^2}{4}$

Area enclosed by two concentric circles: If R and r are radii of two concentric circles, then area enclosed by the two circles = $\pi R^2 - \pi r^2$



$= \pi (R^2 - r^2)$

$= \pi (R + r) (R - r)$

(i) If two circles touch internally, then the distance between their centres is equal to the difference of their radii.

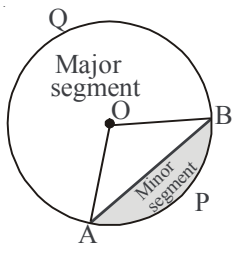
(ii) If two circles touch externally, then distance between their centres is equal to the sum of their radii.

(iii) Distance moved by rotating wheel in one revolution is equal to the circumference of the wheel.

(iv) The number of revolutions completed by a rotating wheel in

$$\text{one minute} = \frac{\text{Distance moved in one minute}}{\text{Circumference of the wheel}}$$

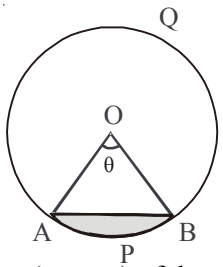
Segment of a Circle: The portion (or part) of a circular region enclosed between a chord and the corresponding arc is called a segment of the circle. In adjacent fig. APB is minor segment and AQB is major segment.



Area of segment APB = Area of the sector OAPB – Area of ΔOAB

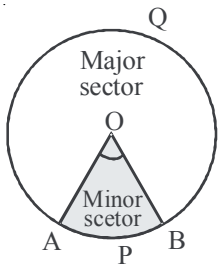
$$= \frac{\theta}{360°} \times \pi r^2 - \frac{1}{2} r^2 \sin\theta \text{ or}$$

$$= \frac{\theta}{360°} \pi r^2 - r^2 \sin\frac{\theta}{2} \cos\frac{\theta}{2}$$



Sector of a circle: The portion (or part) of the circular region enclosed by the two radii and the corresponding arc is called a sector of the circle.

In adjacent figure OAPB is minor sector and OAQB is the major sector.



Area of the sector of angle θ $= \frac{\theta}{360°} \times r^2$

$$= \frac{1}{2} \times \textit{length of arc} \times \textit{radius} = \frac{1}{2} lr$$

Length of an arc of a sector of angle $\theta = \frac{\theta}{360} \times 2\pi r$

(i) The sum of the arcs of major and minor sectors of a circle is equal to the circumference of the circle.

(ii) The sum of the areas of major and minor sectors of a circle is equal to the area of the circle.

(iii) Angle described by minute hand in 60 minutes = 360°

Angle described by minute hand in one minute $= \frac{360°}{60°} = 6°$

Thus minute hand rotates through an angle of 6° in one minute

(iv) Angle described by hour hand in 12 hours = 360°

Angle described by hour hand in one hour $= \frac{360°}{12°} = 30°$

Angle described by hour hand in one minute $= \frac{30°}{60°} = \left(\frac{1}{2}\right)$

Thus, hour hand rotates through an angle of $\left(\frac{1}{2}\right)$ in one minute.

VERY SHORT ANSWER QUESTIONS

1. If the diameter of a semi circular protactor is 14 cm, then find its perimeter.

2. If circumference and the area of a circle are numerically equal, find the diameter of the circle.

3. Find the area of the circle 'inscribed' in a square of side *a* cm.

4. Find the area of a sector of a circle whose radius is *r* and length of the arc is *l*.

5. The radius of a wheel is 0.25 m. Find the number of revolutions it will make to travel a distance of 11 kms.

6. If the area of circle is 616 cm², then what is its circumference?

7. What is the area of the circle that can be inscribe in a square of side 6 cm?

8. What is the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 24 cm and 7 cm?

9. A wire can be bent in the form of a circle of radius 35 cm. If it is bent in the form of a square, then what will be its area?

10. What is the angle subtended at the centre of a circle of radius 6 cm by an arc of length 3π cm?

11. Write the formula for the area of sector of angle θ (in degrees) of a circle of radius *r*.

12. If the circumference of two circles are in the ratio 2:3, what is the ratio of their areas?

13. If the difference between the circumference and radius of a circle is 37 cm, then find the circumference of the circle. (Use $\pi = \frac{22}{7}$)

14. If diameter of a circle is increased by 40%, find by how much percentage its area increases?

15. The hour hand of a clock is 6 cm long. Find the area swept by it between 11:20 am and 11:55 am.

16. What is the diameter of a circle whose area is equal to the sum of areas of two circles of radii 24 cm and 7 cm. (NCERT Exemplar)

17. What is the area of the circel that can be incresed in a square of side 6 cm. (NCERT Exemplar)

18. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in one minute.

19. **Tied the correct Answer**

If the peremeter and the area of a circle are numercally equal, then the radius of the circle is:

(a) 2 units (b) 11 units (c) 4 units (d) 7 units

20. Circumference of a circle of radius r is _____ .

21. Area of a circle of radius s is _____ ,

22. Length of an arc of a sector of a circle with radius *r* and angle θ is _____ .

23. Area of a sector with radius r and angle with degrees measure θ is _____ .

24. Area of segment of a circle = Area of the corresponding sector _____ .

SHORT ANSWER TYPE I QUESTIONS

25. Find the area of a quadrant of a circle whose circumference is 22 cm.

26. What is the angle subtended at the centre of a circle of radius 10 cm by an arc of length 5π cm?

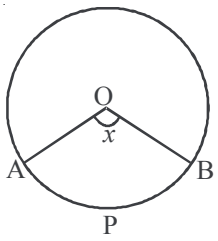
27. If a square is inscribed in a circle, what is the ratio of the area of the circle and the square?

28. Find the radius of semicircle if its perimeter is 18 cm.

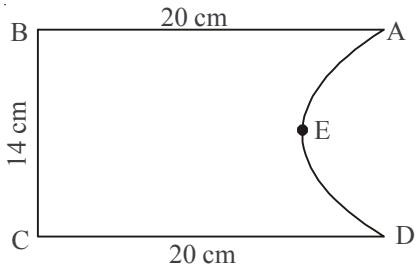
29. If the perimeter of a circle is equal to that of square, then find the ratio of their areas.

30. What is the ratio of the areas of a circle and an equilateral triangle whose diameter and a side are respectively equal?

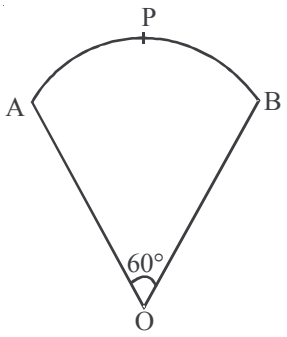
31. In fig., O is the centre of a circle. The area of sector OAPB is $\frac{5}{18}$ of the area of the circle. Find x.



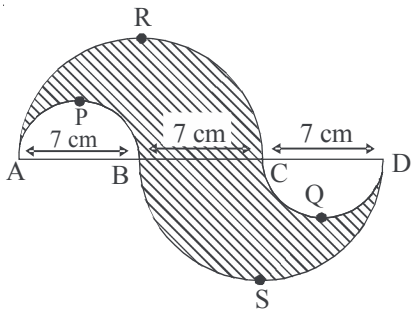
32. Find the perimeter of a given fig, where AED is a semicircle and ABCD is a rectangle. (CBSE, 2015)



33. In fig. OAPBO is a sector of a circle of radius 10.5 cm. Find the perimeter of the sector.



34. In the given fig, APB and CQD are semi circles of diameter 7 cm each, while ARC and BSD are semicircles of diameter 14 cm each. Find the perimeter of the shaded region. (Use $\pi = \frac{22}{7}$) (Delhi, 2011)

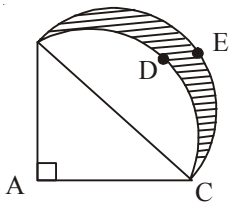


SHORT ANSWER TYPE II QUESTIONS

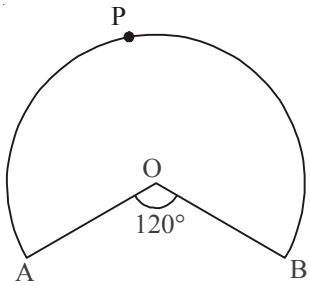
35. Area of a sector of a circle of radius 36 cm is 54π cm^2 . Find the length of the corresponding arc of the sector.

36. The length of the minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time period 6:05 am to 6:40 am.

37. In figure ABCD is a quadrant of a circle of a radius 28 cm and a semi circles BEC is drawn with BC as diameter find the area of shaded region:



38. In fig, OAPB is a sector of a circle of radius 3.5 cm with the centre at O and $\angle AOB = 120°$. Find the length of OAPBO.



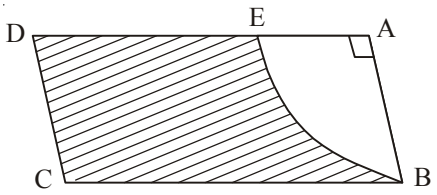
39. Circular footpath of width 2 m is constructed at the rate of ₹ 20 per square meter, around a circular park of radius 1500 m. Find the total cost of construction of the foot path. (Take $\pi = 3.14$)

40. A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60 cm. Calculate the speed of cycle.

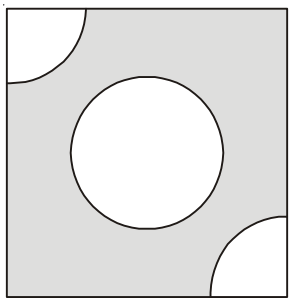
41. In a circle with centre O and radius 4 cm, and of angle 30°. Find the area of minor sector and minor sector AOB. ($\pi = 3.14$)

42. Find the area of the largest triangle that can be inscribed in a semi circle of radius r unit. (NCERT Exempler)

43. Figure ABCD is a trapezium of area 24.5 cm in it AD||BC, $\angle DAB = 90°$, AD = 10 cm, BC= 4cm. If ABE is a quadrant of a circle. Find the area of the shaded region ($\pi = \frac{22}{7}$)

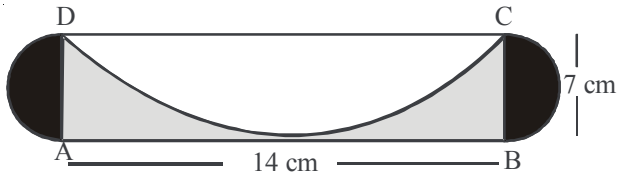


44. From each of the two opposite corners of a square of side 8 cm, a quadrant of a circle of radius 1.4 cm is cut. Another circle of radius 4.2 cm is also cut from the centre as shown in fig. Find the area of the shaded portion. (Use $\pi = \frac{22}{7}$).



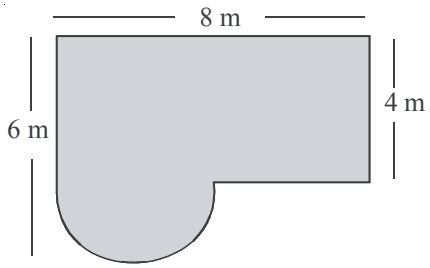
45. A sector of 100° cut off from a circle contains area 70.65 cm². Find the radius of the circle. ($\pi = 3.14$)

46. In fig. ABCD is a rectangle with AB= 14 cm and BC= 7 cm. Taking DC, BC and AD as diameter, three semicircles are drawn. Find the area of the shaded portion.



47. A square water tank has its each side equal to 40 m. There are four semi circular grassy plots all around it. Find the cost of turfing the plot at Rs 1.25 per sq. m. (Use $\pi = 3.14$)

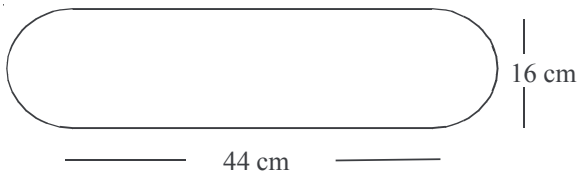
48. Find the area of the shaded region shown in the fig. (NCERT – Exemplar)



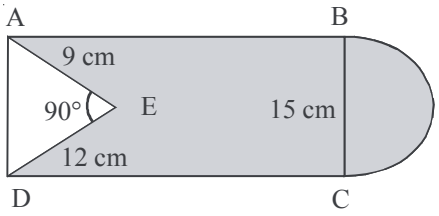
49. Find the area of the minor segment of a circle of radius 21 cm, when the angle of the corresponding sector is 120°.

50. A piece of wire 11 cm long is bent into the form of an arc of a circle subtending an angle of 45° at its centre. Find the radius of the circle.

51. Find the area of the flower bed (with semicircular ends). (NCERT Exampler)

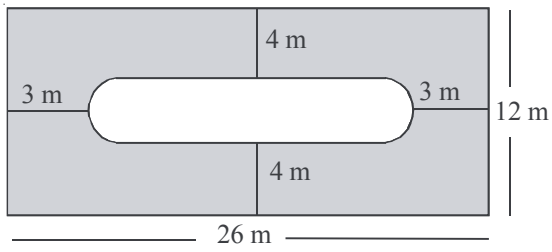


52. In fig. from a rectangular region ABCD with AB= 20 cm, a right triangle AED with AE= 9 cm and DE= 12 cm, is cut off. On the other end, taking BC as diameter, a semi circle is added on outside the region. Find the area of the shaded region.



53. The circumference of a circle exceeds the diameter by 16.8 cm. Find the radius of the circle.

54. Find the area of the shaded region. (NCERT Exampler)



LONG ANSWER TYPE QUESTIONS

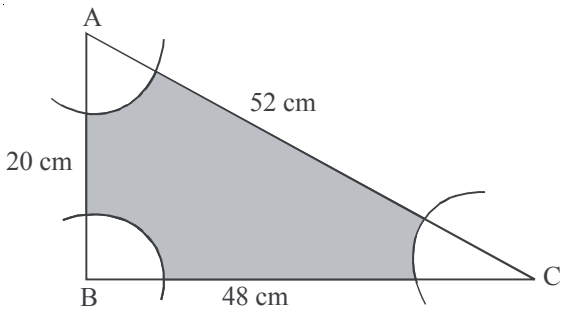
55. Two circles touch externally. The sum of their areas is 130π sq. cm and the distance between their centres is 14 cm. Find the radii of the circles.

56. Three circles each of radius 7 cm are drawn in such a way that each of their touches the other two. Find the area enclosed between the circles. (All India 2010)

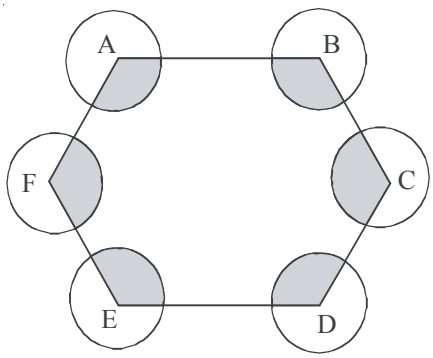
57. Find the number of revolutions made by a circular wheel of area 6.16 m^2 in rolling a distance of 572 m.

58. All the vertices of a rhombus lie on a circle. Find the area of the rhombus, if area of the circle is 2464 cm^2.

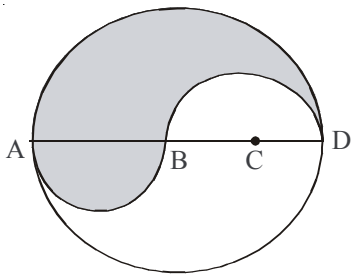
59. With vertices A, B and C of a triangle ABC as centres, arcs are drawn with radius 6 cm each in fig. If AB= 20 cm, BC= 48 cm and CA= 52 cm, then find the area of the shaded region.



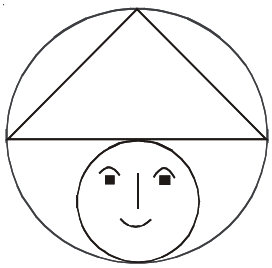
60. ABCDEF is a regular hexagon. With vertices A, B, C, D, E and F as the centres, circles of same radius '*r*' are drawn. Find the area of the shaded portion shown in the given figure.



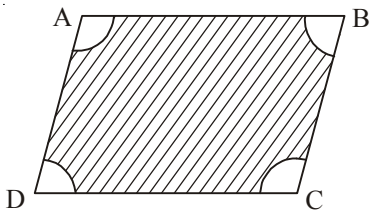
61. ABCD is a diameter of a circle of radius 6 cm. The lengths AB, BC and CD are equal. Semicircles are drawn on AB and BD as diameter as shown in the fig. Find the perimeter and area of the shaded region.



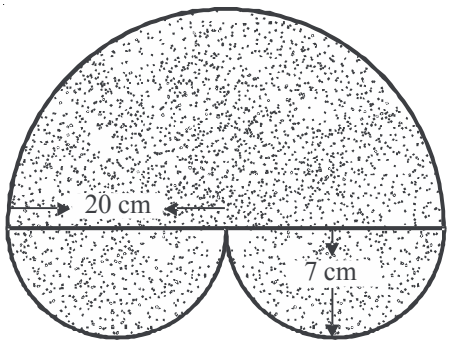
62. A poor artist on the street makes funny cartoons for children and earns his living. Once he made a comic face by drawing a circle within a circle, the radius of the bigger circle being 30 cm and that of smaller being 20 cm as shown in the figure. What is the area of the cap given in this figure?



63. In a given figure ABCD is a trapzium with AB || DC, AB = 18 cm, DC = 32 cm and distance between AB and DC = 14 cm. If arc of equal radii 7 cm with centres A B, C and D have been drawn, then find the area of shaded region.



64. Find the area of the shaded region in the given figure.



ANSWERS AND HINTS

1. $\pi r + d = \frac{22}{7} \times 7 + 14 = 36$ cm

2. $2\pi r = \pi r^2 \Rightarrow 4$ units.

3. Side of the square is equal to diameter of the circle,

$$\pi r^2 = \pi \times \frac{a^2}{4} \quad (\text{side} = a, \text{radius} = \frac{a}{2})$$

4. $l = \frac{\theta}{360^\circ} \times 2\pi r$, Area $= \frac{\theta}{360^\circ} \times \pi r^2 \Rightarrow \frac{l \times \pi r^2}{2\pi r} = \frac{lr}{2}$ sq. units

5. $\frac{\text{distance}}{\text{circumference}} = \frac{11 \times 1000 \times 7 \times 100}{2 \times 22 \times 25} = 7000$

6. $\pi r^2 = 616 \Rightarrow r = 14$ cm or $2\pi r = 88$ cm

7. Side of the square is equal to the diameter of the circle

$\Rightarrow$ r = 3 cm or $\pi r^2 = \pi(3)^2 = 9\pi$ cm^2.

8. $\pi R^2 = \pi r_1^2 + \pi r_2^2 \Rightarrow R = 25$ and diameter = 50 cm.

9. $2\pi r = 2 \times \frac{22}{7} \times 35 = 220$ cm , Side of square $\frac{220}{4} = 55$ cm

Area of square = 55 × 55 = 3025 cm^2

10. $l = \frac{\theta}{360} \times 2\pi r \quad \Rightarrow \quad 3\pi = \frac{\theta}{360} \times 2\pi \times 6 \quad \Rightarrow \quad \theta = 90°$

11. $\frac{\theta}{360} \times 2\pi r$

12. $\frac{2\pi r_1}{2\pi r_2} = \frac{2}{3} \Rightarrow r_1 = \frac{2}{3} r_2$ or $\frac{\pi r_1^2}{\pi r_2^2} = \frac{\left(\frac{2}{3} r_1\right)^2}{r_2^2} = \frac{\frac{4}{9} r_2^2}{r_2^2} = 4:9$

13. $(2\pi r - r) = 37$ or $r = 7$, $2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ cm

14. $\frac{\pi d_1}{\pi d_2} = \frac{100}{140} \Rightarrow \frac{2\pi r_1}{2\pi r_2} = \frac{5}{7}$, $\frac{\pi r_1^2}{\pi r_2^2} = 25 : 49$ $\frac{24}{25} \times 100 = 96\%$

15. $\frac{210 \times 22 \times 6 \times 6}{360 \times 7} = 66$ cm^2 ($\theta = 210°$) (11: 20 to 11: 55 = 35 minutes)

16. $\pi R^2 = \pi r_1^2 + \pi r_2^2 \quad \Rightarrow \quad R = 25$

17. Diameter of the circle = Side of square or 3 cm radius, area of circle = $\pi r^2 = 9\pi$ cm^2

18. 10.27 cm^2

19. 2 units.

20. $2\pi r$

21. πs^2

22. $\frac{\theta}{360} \times 2\pi r$

23. $\frac{\theta}{360} \times \pi r^2$

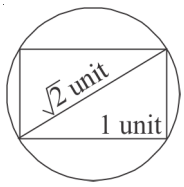
24. Area of the corresponding triangle

25. $2\pi r = 22$, $r = \frac{7}{2}$

Area of quadrant = $\frac{\pi r^2}{4} = \frac{22 \times 7 \times 7}{7 \times 4 \times 2 \times 2} = 9.625$ cm^2

26. $l = \frac{\theta}{360} \times 2\pi r \Rightarrow 5\pi = \frac{\theta}{360} \times 2\pi \times 10 \Rightarrow \theta = 90°$

27.



If side of square is 1 unit by Pythagoras

Diameter or dianotla = $\sqrt{2}$ unit.

Area of square = 1 × 1 = 1 sq units.

Area of Circle = $\pi r^2 = \pi \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{\pi}{2}$

$= \frac{22}{7} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} = \frac{11}{7}$

So, $4 : \pi$ or $11 : 7$

28. $\pi r + 2r = 18$ cm

$\frac{22}{7} r + 2r = 18$

$r\left(\frac{22}{7} + 2\right) = 18$

$r = \frac{7}{2}$ or 3.5 cm

29. $2\pi r = 4$ unit or $\frac{2\pi r}{4 \text{ unit}} = \frac{\text{Perimeter of circle}}{\text{Perimeter of square}}$

$r = \frac{7}{11}$ unit

$\frac{\pi r^2}{1} = \frac{22}{7} \times \frac{7}{11} \times \frac{7}{11} = \frac{14}{11}$ or $14 : 11$

30. Area of equilateral triangle = $\frac{\sqrt{3}}{4} a^2$

Area of circle required = $\pi\left(\frac{a}{2}\right)^2$

$$\text{ratio} = \frac{\frac{\sqrt{3}}{4}a^2}{\pi\left(\frac{a}{2}\right)^2} = \pi : \sqrt{3}$$

31. $\frac{\theta}{360}\pi r^2 = \pi r^2 \times \frac{5}{18}$

$\theta = 100°$

32. 20 cm + 14 cm + 20 cm + πr

20 cm + 14 cm + 20 cm + $\frac{22}{7} \times 7 = 76$ cm

33. $\frac{\theta}{360} \times 2\pi r = \frac{60 \times 2 \times 22 \times 105}{360 \times 7 \times 10} = 11$ cm

Perimeter = 10.5 + 10.5 + 11 cm = 32 cm

34. Perimeter of shaded region = Perimeters of semi circles,

= ARC + APB + BSD + CQD

= $\pi (r_1 + r_2 + r_3 + r_4)$

= $\frac{22}{7}\left[7 + \frac{7}{2} + 7 + \frac{7}{2}\right] = \frac{22}{7} \times 21 = 66$ cm

35. $54\ \pi = \frac{\theta \times \pi \times 36 \times 36}{360}$

$\theta = 15°$

$l = \frac{\theta}{360} \times 2\pi r = \frac{15 \times 2 \times \pi \times 36}{360} = 3\ \pi$

36. Area = $\frac{\theta}{360} \times \pi r^2 = \frac{210 \times 22 \times 5 \times 5}{360 \times 7} = \frac{1650}{36} = 45 \cdot \frac{5}{6}\text{cm}^2$

($\theta = 210°$ in 35 minutes)

37. AC = 28 cm, BC = $28\sqrt{2}$ cm (by Pythagoras).

radius = $14\sqrt{2}$ cm

Shaded region = Area of semicircle – Area of segment BCD

$= \frac{1}{2}\pi(14\sqrt{2})^2 - \frac{90°}{360°} \times \pi\,(28)^2 + \frac{1}{2} \times 28 \times 28$

$= 392\ \text{cm}^2$

38. $l = \frac{240 \times 2 \times 22 \times 35}{360 \times 7 \times 10} = 14.6$

Length of OAPBO = 14.6 + 3.5 + 3.5

= 21.6 cm

39. $\pi\,(r_1^2 - r_1^2) = \pi[(1502)^2 - (1500)^2] \times 20$

$= 3.14\ [(1502)^2 - (1500)^2] \times 20$

= ₹ 3770.51.2

40. Circumference of cycle = $2\pi r$

$= 2 \times \frac{22}{7} \times 30$ cm

= 188.57 cm

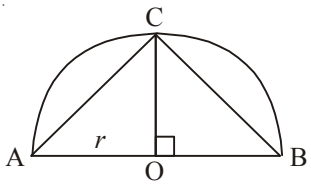
Speed of cycle = $\frac{18857 \times 140 \times 60}{100 \times 100000}$

= 15.84 km/h

41. Area of Minor sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{30}{360} \times 3.14 \times 4 \times 4\ \text{cm}^2$

$= 4.19\ \text{cm}^2$



Area of major sector = $\frac{\theta}{360} \times \pi r^2$

$= \frac{330}{360} \times 3.14 \times 4 \times 4$

$= 46.1 \text{ cm}^2$ (approx)

42. Area of $\Delta = \frac{1}{2}$ base $\times$ height

$= \frac{1}{2} AB \times OC$

$= \frac{1}{2} 2r \times r$

$= r^2$ square unit

43. Let AB = h cm

Area of tropezium $= \frac{1}{2}(AD + BC) \times AB$

$24.5 = \frac{1}{2}(10 + 4) \times 4$

$h = 3.5$ cm

Area of quadrant ABE $= \frac{90°}{360°} \times \pi (3.5)^2$ sq.m

$= 9.625$ sq.m

Area of shaded region $= 24.5 - 9.625$

$= 14.875$ sq. m

44. Area of shaded portion =

Area of square – Area of circle – (Area of 2 quadrauts) or Area of Semicircle.

$= 64 - \frac{22 \times 42 \times 42}{7 \times 10 \times 10} - \frac{22 \times 14 \times 14 \times 1}{7 \times 10 \times 10 \times 2}$

$= 64 - 55.44 - 3.08$

$= 5.48 \text{ cm}^2$

45. $\frac{7065}{100} = \frac{100 \times 314 \times r^2}{360 \times 100}$

$\frac{7065 \times 360}{100 \times 314} = r^2$

$9 = r$

$r = 9$ cm.

46. Area of shaded portion is = One circle and Area of rectangle – semicircle of diamter DC,

$$\text{Area of shaded portion} = \pi r^2\left[AB \times BC - \frac{\pi\left(\frac{DC}{2}\right)^2}{2}\right]$$

$$= \frac{22}{7} \times (3.5)^2 + \left[98 - \frac{22 \times 7 \times 7}{7 \times 2}\right]$$

$$= 38.5 + [98 - 77]$$

$$= 38.5 + 21$$

$$= 59.5 \text{ cm}^2$$

47. Four semicircluar means 2 circles ,

$$\text{Area of 2 circles} = 2\pi r^2$$
$$= 2 \times 3.14 \times 20 \times 20$$
$$= 2512$$
$$= 2512 \times 1.25$$
$$= ₹\ 3140$$

48. Redraw the figure and decide in into well known shapes,

One semi circle + Rectangle

$$\text{Area of shaded region} = l \times b + \frac{\pi r^2}{2}$$

$$= 8 \times 4 + \pi \times \frac{2 \times 2}{2}$$

$$= (32 + 2\pi) \text{ cm}^2$$

49. Area of the segment = Area of sector – Area of Δ

$$\text{Area of sector} = \frac{120}{360} \times \frac{22}{7} \times 21 \times 21 = 462 \text{ cm}^2$$

$$\text{Area of } \Delta = \frac{441}{4}\sqrt{3} \text{ cm}^2 \text{ (NCERT example – 3)}$$

$$\text{Area of segment} = \left(462 - \frac{441}{4}\sqrt{3}\right) \text{ cm}^2$$

$$= \frac{21}{4}\left(88 - 21\sqrt{3}\right) \text{ cm}^2$$

50. $$l = \frac{\theta}{360} \times 2\pi r$$

$$11 = \frac{45}{360}\frac{2 \times 22 \times r}{7}$$

$$14 = r$$

$$r = 14 \text{ cm}$$

51. Flower bed has two semi-circular shapes and one rectangular shape.

$$\text{Area} = l \times b + \pi r^2$$

$$= (44 \times 16 + \pi \times 8 \times 8)$$

$$= (704 + 64\pi) \text{ cm}^2$$

52. Area of shaded region = Rectangle + Semicircle – Triangle

$$= 20 \times 15 + 28.12\ \pi - \frac{1}{2} \times 12 \times 9$$

$$= 334.39 \text{ cm}^2$$

53. $$2\pi r = 2r + 16.8$$

$$2 \times \frac{22}{7} r - 2r = \frac{168}{10} \quad \text{or} \quad 2r\left(\frac{22}{7} - 1\right) = \frac{168}{10}$$

or, $$2r\left(\frac{15}{7}\right) = \frac{168}{10} \quad \text{or} \quad \frac{168 \times 7}{10 \times 2 \times 15} = \frac{1176}{300} = 3.92 \text{ cm}$$

54. Area of shaded region = Area of rectangle – [Area of 2 semicircles + Area of rectangle]

$$= L \times B - \left[2\frac{\pi r^2}{2} + l \times b\right]$$

$$= 26 \times 12 - [\pi \times 2 \times 2 + 16 \times 4]$$

$$= 312 - 4\pi - 64 = (248 - 4\pi) \text{ m}^2$$

55. $$\pi r_1^2 + \pi r_2^2 = 130\ \pi \Rightarrow r_1^2 + r_2^2 + 130 \qquad ...(1)$$

$$\Rightarrow \qquad r_1 + r_2 = 14 \qquad ...(2)$$

Substitude the value of r_1 from (2) in (1) and solve.

$$2r_2^2 - 28\, r_2 + 66 = 0$$

$$r_2^2 - 14r + 33 = 0 \qquad \text{(Neglecting – ve)}$$

$$r = 11 \text{ cm and } r = 3 \text{ cm}$$

56. Area of shaded region = Area of Δ – Area of 3 sectors.

$$\text{area } \Delta = \frac{\sqrt{3}}{4} \times 14 \times 14 = \frac{\sqrt{3}}{4} \times 196 = 49\sqrt{3}$$

$$\text{Area of 3 Sectors} = 3 \times \frac{60}{360} \times \frac{22}{7} \times 7 \times 7 = 77$$

$$= (49\sqrt{3} - 77) \quad \textbf{Ans.}$$

57. $$\pi r^2 = \frac{616}{100} \quad \text{or} \quad r^2 = 1.96 \quad \text{or} \quad r = 1.4 \text{ m}$$

$$2\pi r = 2 \times \frac{22}{7} \times \frac{14}{10} = \frac{616}{100} = 8.8 \text{ m}$$

$$\text{Number of revolution} = \frac{572}{8.8} = 65$$

58. $$\pi r^2 = 2464 \text{ cm}^2$$

$$r = 28 \text{ cm} \quad \text{or} \quad d = 28 + 28 = 56 \text{ cm}$$

$$\text{Area of rhombus} = \frac{1}{2} d_1 d_2 \text{ or } \frac{1}{2} d_2^2 (d_1 = d_2)$$

$$= \frac{1}{2} \times 56 \times 56 = 1568 \text{ cm}^2$$

59. Area of shaded region = Area of Δ – Area of 3 sectors.

$$= \frac{1}{48} \times 20 - \frac{\pi r^2}{360} (\theta_1 + \theta_2 + \theta_3)$$

$$= 480 - \frac{22 \times 6 \times 6}{7 \times 360} (180°)$$

$$= 480 - 56.57$$

$$= 423.43$$

60. $2\pi r^2$ (Area is equal to 2 circles.)

61. $$\text{Perimeter} = \frac{2\pi r_1}{2} + \frac{2\pi r_2}{2} + \frac{2\pi r_3}{2}$$

$$= \left[2 \times \frac{22}{7} \times \frac{6}{2} + 2 \times \frac{22}{7} \times \frac{4}{2} \times 2 \times \frac{22}{7} \times \frac{2}{2}\right]$$

$$= 2 \times \frac{22}{7}[3 + 2 + 1] = \frac{264}{7} = 37.71 \text{ cm}$$

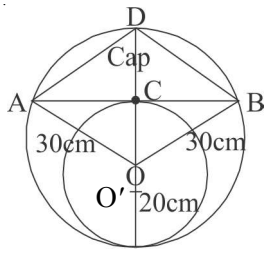
$$\text{Area} = \left[\pi\frac{r_1^2}{2} - \pi\frac{r_2^2}{2} + \pi\frac{r_3^2}{2}\right] = \frac{22}{7}(18 - 8 + 2)$$

$$= 31.71 \text{ cm}^2$$

62. Radius of bigger circle O = 30 cm

Radius of Smaller O′ = 20 cm

Difference of their radii = (30 – 20) = 10 cm



AB is tangent to small circle

Radius = O′C i.e. OD ⊥ AB

$\therefore$ $\angle OCA = 90° = \angle OCB$

In Δ OCA by Phythagoras

$$AC = 20\sqrt{2} \text{ cm}$$

$$AC = CB$$

$\Rightarrow$ $$AB = AC + CB$$

$\Rightarrow$ $$AB = AC + AC = 2\,AC$$

$\Rightarrow$ $$AB = 2 \times 20\sqrt{2} \text{ cm}$$

$$= 40\sqrt{2} \text{ cm}$$

$$CD = \text{Radius of bigger circle-OC}$$

$$= 30–10 = 20 \text{ cm}$$

$$\text{Area of cap} = \frac{1}{2} AB \times CD$$

$$= \frac{1}{2} \times 40\sqrt{2} \times 20 \text{ cm}^2$$

$$= 400\sqrt{2} \text{ cm}^2$$

63. Area of trapezium $= \frac{1}{2} \times h\,(a + b)$

$= \frac{1}{2} \times 14 \times (18 + 32) = 350 \text{ cm}^2$

Area of four sectors $= \frac{\pi r^2}{360} \times (\angle A + \angle B + \angle C + \angle D)$

$= \frac{\pi \times 7 \times 7}{360} \times 360$

$= 49\ \pi \text{ cm}^2$

64. Area of shaded region $= \left(\frac{\pi r_1^2}{2} + \frac{\pi r_2^2}{2} + \frac{\pi r_3^2}{2} \right)$

$= \pi \left(\frac{17 \times 17}{7} + \frac{10 \times 10}{2} + \frac{7 \times 7}{2} \right)$

$= 688.28 \text{ cm}^2$

❐❐❐

PRACTICE-TEST

AREAS RELATED TO CIRCLES

Time : 1 Hr. ***M.M.: 20***

SECTION-A

1. If the circumference of two circles are equal, then what is the ratio between their areas? 1

2. If the diameter of a protactor is 21 cm, then find its perimeter. 1

3. Area of a circle of radius P is _____ . 1

4. Tick the correct answer.

   If the perimeter and the area of a circle are numerically equal then the radius of the circle is 1

   (a) 2 units (b) π units (c) 4 units (d) 7 units

SECTION-B

5. The length of minute hand of a clock is 14 cm. Find the area swept by the mixutre hand in 8 minutes. 2

6. Find the area of a circle whose circumference is 22 cm. 2

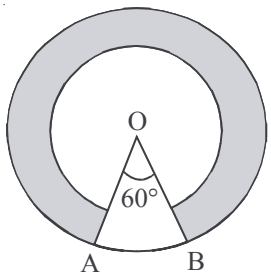
7. Find the area of a quadrant of a circle whose circumference is 44 cm. 2

SECTION-C

8. A horse is tied to a pole with 28 cm long string. Find the area where the horse can graze. 3

9. In fig. two concentric circles with centre O, have radii 21 cm and 42 cm. If $\angle AOB = 60°$ find the area of the shaded region.

   (Use $\pi = \frac{22}{7}$) 3

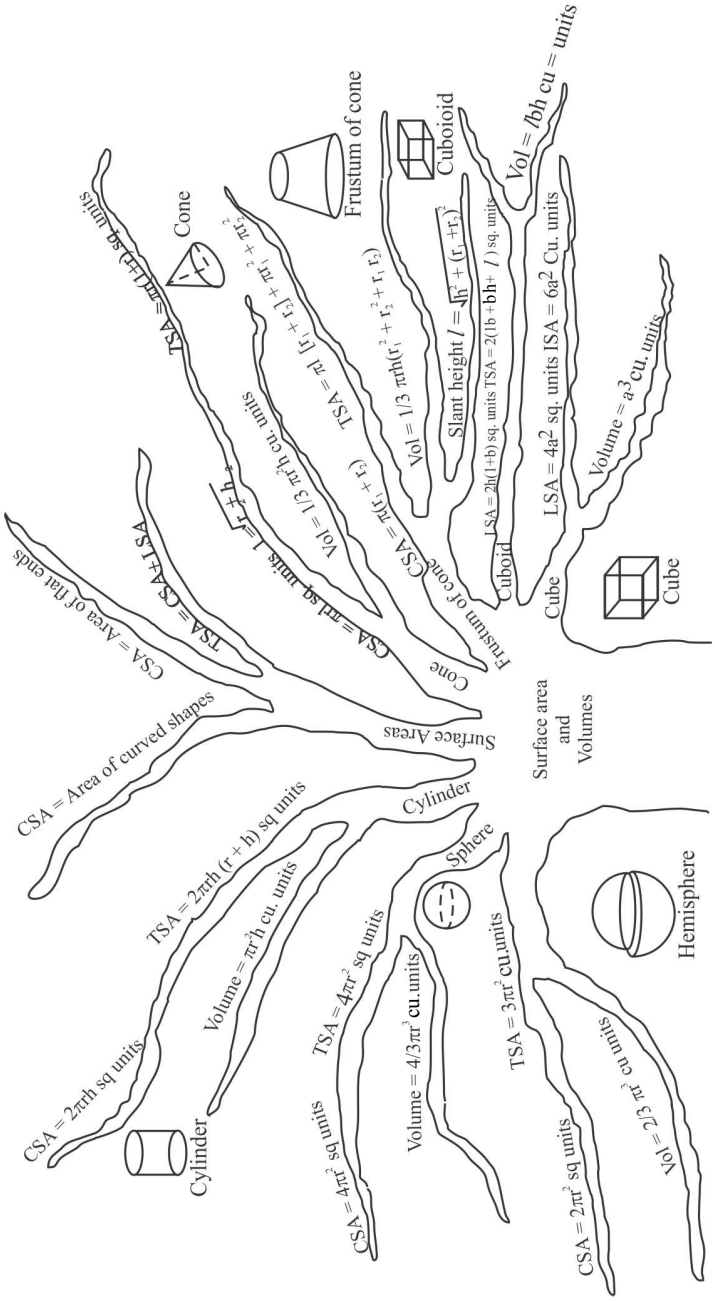


SECTION-D

10. A chord AB of a circle of radius 10 cm makes a right angle at the centre of the circle. Find the area of the minor and major segments. 4

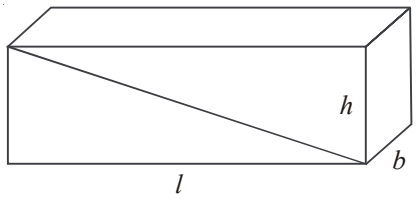
CHAPTER

13 Surface Areas and Volumes



KEY POINTS

1. **Cuboid:** 3-D shapes like a book, a metch box , an almirah, a room etc. are called Cuboid.



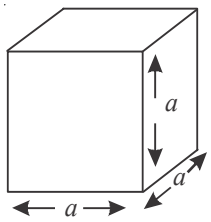
For cuboid length = l, breadth = b, height = h

***Volume* = $l \times b \times h$**

Lateral surface area of solid cuboid = $2h(l + b)$

Total surface area of solid cuboid = $2(lb + bh + hl)$

2. **Cube:** 3-D shapes like ice-cubes, dice, etc. are called cube.



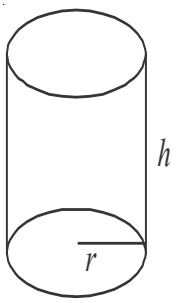
In cube, length = breadth = height = a

***Volume* = a^3**

Lateral surface area of solid cube = $4a^2$

Total surface area of solid cube = $6a^2$

3. **Cylinder**: 3-D shapes like jars, circular pillars, circular pipes, rood rollers etc. are called cylinder.



(a) For right circular cylinder solid, base radius = r, height = h

Volume = $\pi r^2 h$

Lateral surface area of solid cylinder = $2\pi rh$

Total surface area of solid cylinder = $2\pi r\,(r + h)$

(b) For right circular cylinder (Hollow)

external radius = R

internal radius = r

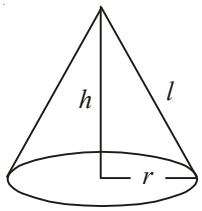
height = h

Volume = $\pi(R^2 - r^2)h$

Curved surface area = $2\pi(R + r)h$

Total surface area = $2\pi(R + r)\,h + 2\pi(R^2 - r^2)$

4. Cone: 3-D shapes like conical tents, ice-cream cone are called Cone.



For right circular cone,

base radius = r

height = h

slant height = l

$l = \sqrt{h^2 + r^2}$

Volume $= \frac{1}{3}\ \pi r^2 h$

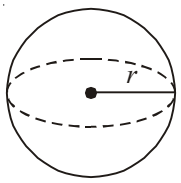
Curved surface area of solid cone = πrl

Total surface area of solid cone = $\pi r\,(r + l)$

It may be noted that if radius and height of a cone and cylinder are same then

3 × volume of a cone = volume of right circular cylinder

5. **Sphere**: 3-D shapes like cricket balls, footballs etc. are called sphere.



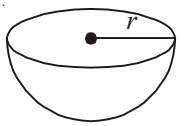
(a) For sphere : Radius = r

Volume = $\frac{4}{3}\pi r^3$

surface area = $4\pi r^2$

(b) For Hemisphere (solid): Radius = r

Volume = $\frac{2}{3}\pi r^3$



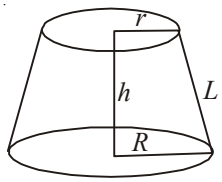
Curved surface area = $2\pi r^2$

Total surface area = $3\pi r^2$

6. **Frustum**: When a cone is cut by a plane parallel to the base of the cone, then the portion between the plane and the base is called the frustum of the cone.

Example = Turkish Cap

For a frustum of cone:



Base radius = R

Top radius = r

Height = h

slant height = l

$$l = \sqrt{h^2 + (R - r)^2}$$

$$\text{volume} = \frac{1}{3}\pi h(r^2 + R^2 + Rr)$$

Curved surface area (solid frustum) = $\pi l(R + r)$

Total surface area (solid frustum) = $\pi l(R + r) + \pi(R^2 + r^2)$

VERY SHORT ANSWER TYPE QUESTIONS

1. Match the following:

Column I	**Column II**
(a) Surface area of a sphere	(i) $2\pi rh$
(b) Total surface area of a cone	(ii) $\frac{1}{2}\pi r^2 h$
(c) Volume of a cuboid	(iii) $2\pi r\{r + h)$
(d) Volume of hemisphere	(iv) $\frac{1}{3}\pi h(r^2 + R^2 + rR)$
(e) Curved surface area of a cone	(v) πr (r + 1)
(f) Total surface area of hemisphere	(vi) $l \times b \times h$
(g) Curved surface area of a cylinder	(vii) $\frac{2}{3}\pi r^3$
(h) Volume of a cone	(viii) πrl
(i) Total surface area of a cylinder	(ix) 3 πr^2
(j) Volume of a frustum of a cone	(x) $4\pi r^2$

2. Fill in the Wanks:

(*i*) The total surface area of cuboid of dimension $a \times a \times$ b is._____.

(*ii*) The volume of right circular cylinder of base radius r and height 2r is _____.

(iii) The total surface area of a cylinder of base radius r and height h is_____.

(*iv*) The curved surface area of a cone of base radius r and height h is_____.

(v) If the height of a cone is equal to diameter of its base, the volume of cone is_____ .

(*vi*) The total surface area of a hemisphere of radius r is_____.

(vii) The lateral surface area of a hollow cylinder of outer radius R, inner radius r and height h is

(*viii*) If the radius of a sphere is doubled, its volume becomes_____times the volume of original sphere.

(ix) If the radius of a sphere is halved, its volume becomes_____times the volume of original sphere. (NCERT Exemplar)

3. Write 'True' or 'False' in the following:

(*i*) Two identical solid hemispheres of equai base radius r are stuck together along their bases. The total surface area of the combination is $6\pi r^2$.

(*ii*) A solid cylinder of radius *r* and height h is placed over other cylinder of same height and radius. The total surface area of the shape so formed is $4\pi rh + 4\pi r^2$.

(iii) A solid cone of radius *r* and height h is placed over a solid cylinder having same base radius and height as that of a cone. The total surface area of the combined. $\pi r(\sqrt{r^2 + h^2 + 3r + 2h})$

(iv) A solid ball is exactly fitted inside the cubical box of side a. The volume of the ball iis $\frac{4}{3}\pi a^2$.

(v) The volume of the frustum of a cone is $\frac{1}{3}\pi h(r_1^2 + r_2^3 + r_1 r_2)$. where *h* is vertical height of the frustum and r_1; r_2 are the radii of the ends.

4. The total surface area of a solid hemisphere of radius *r* is

(a) πr^2 (b) $2\pi r^2$ (c) $3\pi r^2$ (d) $4\pi r^2$

5. The volume and the surface area of a sphere are numerically equal, then the radius of sphere is

(a) 0 units (b) 1 units (c) 2 units (d) 3 units

6. A cylinder, a cone and a hemisphere are of the same base and of the same height. The ratio of their volumes is

(a) 1:2:3 (b) 2:1:3 (c) 3:1:2 (d) 3:2:1

7. A solid sphere of radius '*r*' is melted and recast into the shape of a solid cone of height '*r*'. Then the radius of the base of cone is

(a) 2r (b) r (c) 4r (d) 3r

8. Three solid spheres of diameters 6 cm, 8 cm and 10 cm are melted to form a single solid sphere. The diameter of the new sphere is

(a) 6 cm (b) 4.5 cm (c) 3 cm (d) 12 cm

9. The radii of the ends of a frustum of a cone 40 cm high are 38 cm and 8 cm. The slant height of the frustum of cone is

(a) 50 cm (b) $10\sqrt{7}$ cm

(c) 60.96 cm (d) $4\sqrt{2}$ cm

10. A metallic spherical shell of internal and external diameters 4 cm and 8 cm, respectively is melted and recast into the form of a cone of base diameter 8 cm. The height of the cone is:

(a) 12 cm (b) 14 cm

(c) 15 cm (d) 18 cm

11. A solid piece of iron in the form of a cuboid of dimensions 49 cm × 33 cm × 24 cm, is moulded to form a solid sphere. The radius of the sphere is

(a) 21 cm (b) 23 cm

(c) 25 cm (d) 19 cm

12. A shuttle cock used for playing badminton has the shape of the combination of (NCERT Exemplar)

(a) A cylinder and a sphere (b) a cylinder and a hemisphere

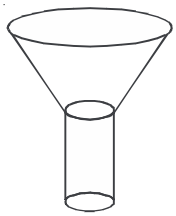
(c) a sphere and a cone (d) frustum of a cone and hemsphere

13. The radii of the top and bottom of a bucket of slant height 45 cm are 28 crn and 7 cm, respectively. The curved surface area of the bucket is (NCERT Exemplar)

(a) 4950 cm^2 (b) 4951 cm^2

(c) 4952 cm^2 (d) 4953 cm^2

14. What geometrical shapes is a "FUNNEL" combination of?



15. What geometrical shapes is a cylindrical "PENCIL" sharped at one edge combination of?

16. What geometrical 3-D shapes is a "GLASS (tumbler)"?

17. What geometrical shapes is a "GILLI" in gilli-danda game combination of?

18. A solid shape is converted from one form to another. What is the change in its volume?

19. What cross-section is made by a cone when it is cut parallel to its base?

20. Find total surface area of a solid hemi-sphere of radius 7cm.

21. Volume of two spheres is in the ratio 64 : 125. Find the ratio of their surface areas.

22. A cylinder and a cone are of same base radius and of same height. Find the ratio of the volumes of cylinder to that of the cone.

23. A solid sphere of radius r is melted and recast into the shape of a solid cone of height *r*. Find radius of the base of the cone.

24. If the volume of a cube is 1331 cm^3, then find the length of its edge.

SHORT ANSWER TYPE QUESTION (TYPE-I)

25. How many cubes of side 2 cm can be cut from a cuboid measuring (16cm×12cm×10cm).

26. Find the height of largest right circular cone that can be cut out of a cube whose volume is 729 cm^3.

27. Two identical cubes each of volume 64 cm^3 are joined together end to end. What is the surface area of the resulting cuboid?

28. Twelve solid spheres of the same sizes are made by melting a solid metallic cylinder of base diameter 2 cm and height 16cm. Find the radius of each sphere.

29. The diameters of the two circular ends of the bucket are 44 cm and 24 cm. The height of the bucket is 35cm. Find the volume of the bucket.

SHORT ANSWER TYPE QUESTION (TYPE-II)

30. A bucket is in the form of a frustum of a cone and hold 28.490 litres of water. The radii of the top and bottom are 28 cm and 21 cm respectively. Find the height of the bucket.

31. Three cubes of a metal whose edge are in the ratio 3:4:5 are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edge of three cubes.

32. Find the depth of a cylindrical tank of radius 10.5 cm, if its capacity is equal to that of a rectangular tank of size 15 cm × 11 cm × 10.5 cm.

33. A cone of radius 8cm and height 12cm is divided into two parts by a plane through the mid-point of its axis parallel to its base. Find the ratio of the volumes of the two parts.

34. A petrol tank is a cylinder of base diameter 28cm and length 24cm filted with conical ends each of axis length 9cm. Determine the capacity of the tank.

35. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/hour. How much area will it irrigate in 30 minutes; if 8 cm standing water is needed? (NCERT CBSE 2019)

36. A solid is in the form of a cylinder with hemispherical ends. The total height of the solid is 20 cm and the diameter of the cylinder is 7 cm. Find the total volume of the solid. (Use $F = \frac{22}{7}$) CBSE 2019

37. Two spheres of same metal weight 1 Kg and 7 Kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the new sphere. CBSE 2019

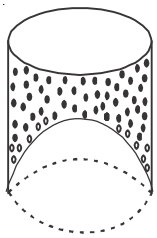
38. A cone of height 24 cm and radius of base 6 cm is made up of modeling clay, A child reshapes it in the form of a sphere. Find the radius of the sphere and hence find the surface area of this sphere. (NCERT CBSE 2019)

39. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field which is 10 m in diameter and 2 m deep. If water flows through pipe at the rate of 3 Km/hr, in How much time will the tank be filled?

(NCERT CBSE 2019)

40. A juice seller was serving his customers using glasses as shown in figure. The inner diameter of the cylindrical glass was 5 cm but bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent and actual capacity of the glass. { Use π] = 3.14}

(NCERT CBSE 2019, 2009)



41. A girl empties a cylindrical bucket full of sand, of base radius 18 crn and height 32 cm on the floor to form a conical heap of sand. If the height of this conical heap is 24 cm, then find its slant height correct to one place of decimal.

(CBSE 2019)

42. Water is flowing at the rate of 5 km/hour through a pipe of diameter 14 cm into a tank with rectangular base which is 50 m long and 44 m wide. Find the time in which the level of water tank rises by 7 cm. (Use $\pi = \frac{22}{7}$ } (CBSE 2019)

43. A field is in the form of rectangle of length 20 m and width 14 m, A 10 m deep well of diameter 7 m is dug in one corner of the field and the earth taken out of the well is spread evenly over the remaining part of the field. Find the rise in the level of the field. (Use $\pi = \frac{22}{7}$) (CBSE 2019)

LONG ANSWER TYPE QUESTIONS

44. A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm^2. The radii of the top and bottom of the circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of the metal sheet used in making it. (Use $\pi = 3.14$) (CBSE 2019)

45. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8 gm mass. (Use Π = 3.14) (NCERT CBSE 2019)

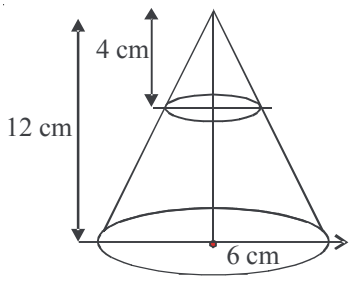
46. A right cylindrical container of radius 6 cm and height 15 cm is full of ice-cream, which has to be distributed to 10 children in equal cones having hemispherical shape on the top. If the height of the conical portion is four times its base radius, find the radius of the ice-cream cone.(CBSE 2019)

47. A container opened at the top and made up of a meta! sheet, is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the container, at the rate of ? 50 per litre. Also find the cost of meta! sheet used to make the container, if it costs ₹ 10 per 100 cm^2 (Take $\pi = 3.14$).

(NCERT CBSE 2019)

48. An open metallic bucket is in the shape of a frustum of a cone, If the diameters of the two circular ends of the bucket are 45 cm and 25 cm and the vertical height of the bucket is 24 cm, find the area of the metallic sheet used to make the bucket. Also find the volume of the water it can hold. { Use $\Pi = \frac{22}{7}$).

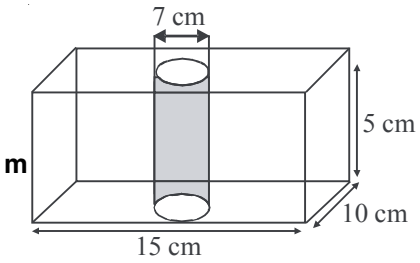
49. In the given figure, from the top of a solid cone of height 12cm and base radius 6cm, a cone of height 4 cm is removed by a plane parallel to the base. Find the total surface area of the remaining solid.(Use $\pi = \frac{22}{7}$ and $\sqrt{5} = 2.236$) (CBSE – 2015)



50. A solid wooden toy is in the form of a hemi-sphere surmounted by a cone of same radius. The radius of hemi-sphere is 3.5cm and the total wood used in the making of toy is $166\frac{5}{6}$ cm^3. Find the height of the toy. Also, find the cost of painting the hemi-spherical part of the toy at the rate of ₹ 10 per cm^2.

(use $\pi = \frac{22}{7}$). (CBSE, 2015)

51. In the given figure, from a cuboidal solid metalic block of dimensions 15 cm × 10 cm × 5 cm a cylindrical hole of diameter 7cm is drilled out. Find the surface area of the remaining block. (Use $\pi = \frac{22}{7}$).(CBSE – 2015)



52. A solid toy is the form of a right circular cylinder with a hemispherical shape at one end and a cone at the other end. Their diameter is 4.2 cm and the heights of the cylindrical and conical portions are 12 cm and 7 cm respectively. Find the volume of the toy.

53. A tent is in the shape of a right circular cylinder upto a height of 3m and conical above it. The total height of the tent is 13.5 m and radius of base is 14 m. Find the cost of cloth required to make the tent at the rate of ₹ 80 per sq. m.

54. The rain water from a roof 22m × 20m drains into a cylindrical vessel having diameter of base 2m and height 3.5m. If the vessel is just full, find the rainfall in cm.

55. The difference between outer and inner curved surface areas of a hollow right circular cylinder, 14 cm long is $88cm^2$. If the volume of the metal used in making the cylinder is $176cm^3$. Find the outer and inner diameters of the cylinder.

ANSWERS AND HINTS

1. (a) (x) $4\pi r^2$

(b) (v) $\pi r (r + l)$

(c) (vi) $l \times b \times h$

(d) (vii) $\frac{2}{3}\pi r^3$

(e) (viii) πrl

(f) (ix) $3\pi r^2$

(g) (i) $2\pi rh$

(h) (ii) $\frac{1}{3}\pi r^2 h$

(i) (iii) $2\pi r(r + h)$

(j) (iv) $\frac{1}{3}\pi h(r^2 + R^2 + rR)$

2. (i) $2a^2 + 4ab$ (ii) $2\pi r^3$

(iii) $2\pi r(r + h)$ (iv) $\pi r\sqrt{r^2 + h^2}$

(v) $\frac{2}{3}\pi r^3$ (vi) $3\pi r^2$

(vii) $2\pi h(R + r)$ (viii) 8

(ix) $\frac{1}{8}$

3. (i) False (ii) False

(iii) False (iv) False

(v) True

4. (c) $3\pi r^2$ **5.** (d) 3 units

6. (c) 3 : 1 : 2 **7.** (a) 2r

8. (d) 12 cm **9.** (a) 50 cm

10.(b) 14 cm **11.** (a) 21 cm

12.(d) Frustem of a cone and a hemisphere **13.** (a) 4950 cm^2

14. Cylinder and frustem of a cone **15.** Cylinder and cone

16. Frustum of a cone **17.** Cylinder with conical ends

18. Remains unchanged **19.** Circle

20. 462 cm^2 **21.** 16 : 25

22. 3 : 1 **23.** 2r

24. 11 cm

25. No. of cubes = $\frac{16 \times 12 \times 10}{2 \times 2 \times 2} = 240$

26. Side of cube = $\sqrt[3]{729} = 9$cm

Height of largest cone = Side of cube = 9 cm

27. Side of cube = $\sqrt[3]{64} = 4$ cm

Length, breadth and height of new cuboid is 8 cm, 4 cm and 4 cm respectively.

Surface area of cuboid = $2[8 \times 4 + 4 \times 4 + 4 \times 8] = 160\ cm^2$

28. Volume of 12 solid sphere = Volume of solid cylinder

$12 \times \frac{4}{3}\pi r^3 = \pi(1)^2 \times 16$

$r^3 = 1$

$r = 1$ cm

29. Volume of bucket = $\frac{1}{3} \times \frac{22}{7} \times 35\ [(22)^2 + (12)^2 + 22 \times 12]$

$= 32706\frac{2}{3}$ cm^3

30. Volume of bucket = 28490 cm^3

$\frac{1}{3} \times \frac{22}{7} \times h\ [(28)^2 + (21)^2 + 28 \times 21] = 28490$

$h = 15$ cm

31. Let the edges of three cubes be 3x cm, 4x cm and 5x cm.

Volume of single cube = Sum of volume of three cubes

$(\text{Side})^3 = (3x)^3 + (4x)^3 + (5x)^3$

Side = 6x cm

Diagonal of single cube = $12\sqrt{3}$

$\sqrt{3}\ (6x) = 12\sqrt{3}$

$x = 2$

Hence edges of three cubes are 6 cm, 8 cm and 10 cm

32. Capacity of cylindrical tank = Capacity of rectangular tank

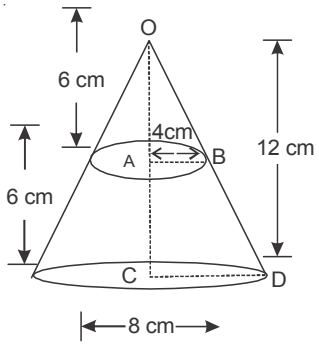
$\frac{22}{7} \times (10.5)^2 \times h = 15 \times 11 \times 10.5$

$h = 5$ cm

33. $\Delta OAB \sim \Delta OCD$

$\frac{AB}{CD} = \frac{OA}{OC}$

$AB = 4$ cm

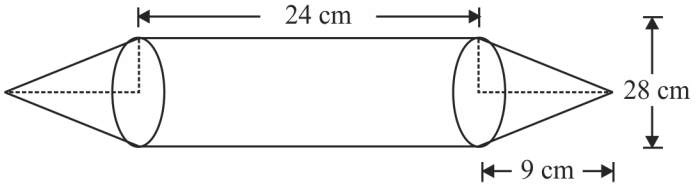


$$\frac{\text{Volume of conical part}}{\text{Volume of frustum part}} = \frac{\frac{1}{3}\pi(4)^2 \times 6}{\frac{1}{3}\pi \times 6[(8)^2 + (4)^2 + 8 \times 4]} = \frac{1}{7}$$

$\therefore$ required ratio is 1 : 7 or 7 : 1

34. Capacity of tank = Volume of cylindrical part + 2 × Volume of conical part

$= 18480 \text{ cm}^2$



35. Length of canal covered in 30 mins = 5000 m

$\therefore$ Volume of water flown in 30 mins

$= 6 \times 1.5 \times 5000 \text{ m}^3$

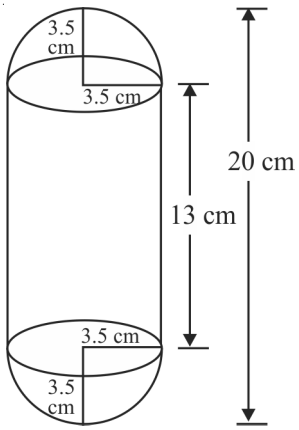
$$\text{Area irrigated} = \frac{6 \times 1.5 \times 5000}{0.08} = 562500 \text{ m}^2$$

36. Height of cylinder = 20 – 3.5 – 3.5 = 13 cm

Volume of solid = Volume of cylindrical part + 2 × Volume of hemispherical part

$$= \frac{22}{7} \times (3.5)^2 \times 13 + 2 \times \frac{2}{3} \times \frac{22}{7} (3.5)^3$$

$$= 680\frac{1}{6} \text{ cm}^3$$



37. Radius of first sphere = 3 cm

Let density of metal be d kg/cm^3

$$\therefore \quad \frac{4}{3}\pi(3)^3 \times d = 1 \qquad ...(1)$$

Let radius of second sphere be rcm.

$$\therefore \quad \frac{4}{3}\pi(r)^3 \times d = 7 \qquad ...(2)$$

From (1) and (2), we have

$$r^3 = 7(3)^3$$

Let the radius of new spnere by R cm.

A.T.Q

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi(3)^3 + \frac{4}{3}\pi r^3$$

$$R^3 = (3)^3 + 7(3)^3$$

$$R = 6 \text{ cm}$$

$\therefore$ Diameter of new sphere $= 2 \times 6 = 12$ cm.

38. Volume of sphere = Volume of cone

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi(6)^2 \times 24$$

$$r = 6 \text{ cm}$$

Surface area of sphere $= 4 \times \pi \times (6)^2 = 144\ \pi$ cm^2

39. $$\text{Time to fill tank} = \frac{\text{Volume of cylindrical tank}}{\text{Volume of water flown in 1 hour}}$$

$$= \frac{\pi(50)^2 \times 2}{\pi\left(\frac{1}{10}\right)^2 \times 3000} = 100 \text{ minutes or 1 hour 40 minutes.}$$

40. Apparent capacity $= 3.14 \times \left(\frac{5}{2}\right)^2 \times 10 = 196.25$ cm^3.

Actual capacity = Volume of cylindrical part – Volume of hemispherical part

$$= 196.25 - \frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^3$$

$= 163.54$ cm^3 approx

41. Volume of conical heap = Volume of cylindrical bucket

$$\frac{1}{3}\pi r^2 \times 24 = \pi(18)^2 \times 32$$

$$r = 36 \text{ cm}$$

Slant height, $l = \sqrt{(36)^2 + (24)^2} = 43.2$ cm

42. Volume of raised water in tank $= 50 \times 44 \times \frac{7}{100} = 154$ m^3

Volume of water flown in 1 hr $= \frac{22}{7} \times \left(\frac{7}{100}\right)^2 \times 5000 = 77$ m^3

$$\text{Time taken} = \frac{154}{77} = 2 \text{ hours}$$

43. $\text{Rise in level} = \dfrac{\text{Earth taken out}}{\text{Area of the remaining part of field}}$

$$= \frac{\frac{22}{7}\times\left(\frac{7}{2}\right)^2\times 10}{\left[20\times 14-\frac{22}{7}\times\frac{7}{2}\times\frac{7}{2}\right]} = 1.5 \text{ m approx.}$$

44. Volume of bucket = 12308.8 cm^3

$$\frac{1}{3}\times 3.14\times h\,[(20)^2 + (12)^2 + 20 \times 12] = 12308.8$$

$$h = 15 \text{ cm}$$

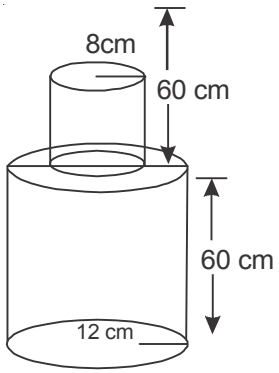
$$l = \sqrt{(15)^2+(20-12)^2} = 17 \text{ cm}$$

Surface area of metal sheet used

$$= 3.14 \times 17 \times (20 + 12) + 3.14 \times (12)^2$$

$$= 2160.32 \text{ cm}^2$$

45.



Volume of solid = $3.14 \times (12)^2 \times 220 + 3.14 \times (8)^2 \times 60$

$$= 111532.8 \text{ cm}^3$$

$$\text{Mass of the pole} = 111532.8 \times \frac{8}{1000} \text{ kg}$$

$$= 892.2624 \text{ kg}$$

46. Let radius of conical section be r cm.

$\therefore$ Height of conical section be 4r cm.

According to the question

$10 \times$ Volume of ice-cream in 1 cone = Volume of cylindrical container

$$10 \times \left[\frac{1}{3}\pi r^2 \times 4r + \frac{2}{3}\pi r^3\right] = \pi(6)^2 \times 15$$

$$r = 3 \text{ cm}$$

47. Volume of the container $= \frac{3.14 \times 16}{3}[(20)^2 + (8)^2 + 20 \times 8]$

$= 10450 \text{ cm}^3$ approx.

$= 10.45$ litres

Cost of milk $= 10.45 \times 50 = ₹ 522.50$

Slant height $= \sqrt{(16)^2 + (20-8)^2} = 20$ cm

Surface area of container

$= 3.14 \times 20\ (20 + 8) + 3.14 \times (8)^2$

$= 1959.36 \text{ cm}^2$

Cost of metal sheet $= \frac{10}{100} \times 1959.36 = ₹ 195.94$

48. Slant height $= \sqrt{(24)^2 + \left(\frac{45}{2} - \frac{25}{2}\right)^2} = 26$ cm

Surface area of bucket $= \frac{22}{7} \times 26 \times \left(\frac{45}{2} + \frac{25}{2}\right) + \frac{22}{7} \times \frac{25}{2} \times \frac{25}{2}$

$= 3351.07 \text{ cm}^2$ approx.

Volume $= \frac{1}{3} \times \frac{22}{7} \times 24 \times \left[\left(\frac{45}{2}\right)^2 + \left(\frac{25}{2}\right)^2 + \frac{45}{2} \times \frac{25}{2}\right]$

49. Radii of frustum are 6 cm and 2 cm.

Height of frustum $= 12.4 = 8$ cm

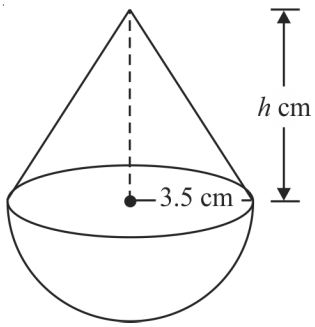
Slant height $= \sqrt{(8)^2 + (6.2)^2} = 4\sqrt{5}$ cm

Total surface area of frustum

$= \frac{22}{7} \times 4 \times 2.236 \times [6 + 2] + \frac{22}{7} \times (6)^2 + \frac{22}{7} \times (2)^2$

$= 350.592 \text{ cm}^2$ approx.

50. Volume of toy = $\frac{1001}{6}$ cm^3



$$\frac{2}{3}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^3+\frac{1}{3}\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2\times h = \frac{1001}{6}$$

$$h = 6\text{ cm}$$

Area of hemispherical part of toy

$$= 2\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2 = 77\text{ cm}^2$$

Cost of painting = $77 \times 10 = ₹\ 770$

51. Surface of the remaining block = TSA of cuboidal block + CSA of cylinder
Area of two circular bases

$$= 2(15\times 10 + 10\times 5 + 15\times 5) + 2\times\frac{22}{7}\times\frac{7}{2}\times 5 - 2\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2$$

$$= 583\text{ cm}^2$$

52. Volume of toy = Volume of cylindrical part + Volume of hemispherical part
+ Volume of conical part

$$= \frac{22}{7}\times(2.1)^2\times 12+\frac{1}{3}\times\frac{22}{7}\times(2.1)^2\times 7+\frac{2}{3}\times\frac{22}{7}\times(2.1)^3$$

$$= 218.064\text{ cm}^3$$

53. Slant height = $\sqrt{(14)^2+(10.5)^2}$ = 17.5 m

$$\text{Surface area of tent} = 2\times\frac{22}{7}\times 3\times 14+\frac{22}{7}\times 14\times 17.5$$

$$= 1034\text{ m}^2$$

Cost of cloth = $1034 \times 80 = ₹\ 82720$

54. $$\text{Rainfall} = \frac{\text{Volume of cylindrical vessel}}{\text{Area of roof}}$$

$$= \frac{\frac{22}{7}\times(1)^2\times 3.5}{22\times 20} = \frac{1}{40}\text{ m}$$

$$= \frac{1}{40}\times 100\text{ cm} = 2.5\text{ cm}$$

55. Let inner and outer radius of hallow cylinder be r cm and R cm respectively.

Difference between Outer and Inner CSA = 88 cm^2

$$2 \times \frac{22}{7} \times 14 \times [R - r] = 88$$

$$R - r = 1 \qquad ...(1)$$

Volume of hollow cylinder = 176 cm^3

$$\frac{22}{7} \times 14 \times [R^2 - r^2] = 176$$

$$R^2 - r^2 = 4$$

$$(R - r)(R + r) = 4$$

$$R + r = 4 \qquad ...(2) \ [\because \text{ from (1)}]$$

From (1) and (2), we get

$$R = 2.5 \text{ cm and } r = 1.5 \text{ cm}$$

$\therefore$ Outer and inner diameter are 5 cm and 3 cm respectively.

PRACTICE-TEST

SURFACE AREAS AND VOLUMES

Time : 1 Hr. ***M.M.: 20***

SECTION-A

1. The total surface area of a hemisphere of radius r is 1

2. Which two geometrical shapes are obtained by cutting a cone parallel to its base? 1

(a) a cylinder and a cone (b) a cone and a hemisphere

(c) a sphere and a cone (d) frustum of a cone and a cone

3. The radius (in cm) of the largest right circular cone that can be cut out from a cube of edge 4.2 cm is 1

(a) 4.2 (b) 2.1

(c) 8.4 (d) 1.05

4. The volume of a cube is 1000 cm^3. Find the length of the side of the cube. 1

SECTION-B

5. The radii of the ends of a frustum of a cone 45 cm high are 28 cm and 7 cm. Find its volume. 2

6. A solid sphere of radius 10.5 cm is melted and recast into smaller solid cones, each of radius 3.5 cm and height 3 cm. Find the number of cones so formed. 2

7. A cube and a sphere have equal total surface area. Find the ratio of the volume of sphere and cube. 2

SECTION-C

8. A vessel is in the form of an inverted cone. Its height is 8 cm and the radius of its top, which is open, is 5 cm. It is filled with water up to brim. When lead shots, each of which is a sphere of radius 0.5 cm are dropped in to the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel. 3

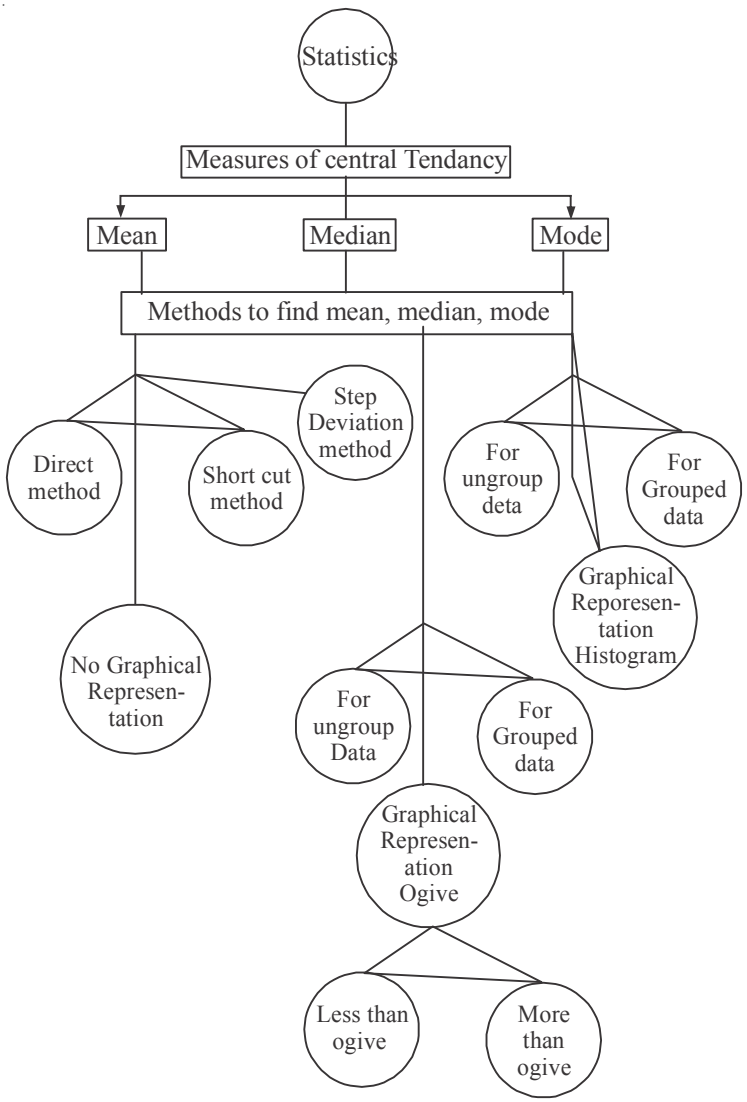
9. A large right circular cone is made out of a solid cube edge 9 cm. Find the volume of the remaining solid. 3

SECTION-D

10. In a hospital, used water is collected in a cylindrical tank of diameter 2 m and height 5 m. After recycling, this water is used to irrigate a park of hospital whose length is 25 m and breadth is 20 m. If tank is filled completely then what will be the height of standing water used for irrigating the park? 4

CHAPTER

14 Statistics



LIST OF FORMULES

1. **Mean** $\bar{x}$

(a) For raw data $\bar{x} = \frac{\sum x_i}{n} = \frac{x_1 + x_2 + ... + xn}{n}$

$$i.\ e.\ \bar{x} = \frac{sum\ of\ observations}{no\ of\ observations}$$

(b) For Grouped data

(i) If small calculation then we apply Direct method

$$\bar{x} = \frac{\sum f_i\, x_i}{\sum f_i}$$

(ii) If calculations are tedius or observations are large then we apply short cut/ Assumed Mean method or step Deviation method

Short cut/Assumed Mean Method

$$\bar{x} = a + \frac{\sum f_i d_i}{\sum f_i}\ ,\ a \rightarrow \text{assumed mean}$$

$$d_i = x_i - a$$

Step Deviation Method

$$\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h\ ,\ u_i = \frac{d_i}{h}\ ,\ h \rightarrow \text{class size}$$

2. **Median**

(a) For ungrouped data we first arrange data in ascending or descending order.

Count number of times say n. If *n* is odd then Median = $\left(\frac{n+1}{2}\right)^{th}$ observation

It *n* is even then Median = $\frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$ obsevation

(b) For grouped data

$$\text{Median} = l + \frac{\left(\frac{n}{2} - cf\right)}{f_i} \times h .$$

(3) $\text{Mode} = l + \frac{(f_1 - f_o)}{(2f_1 - f_o - f_2)} \times h$ (For grouped data)

For ungrouped data mode is the most frequent observation.

NOTES:

1. Empirical relationship between three measures of central tendency:

mode = 3 median – 2 mean.

2. It class interval is discontinuous then make it continuous by subtracting 0.5 from Lower Limit and adding 0.5 to upper limit.

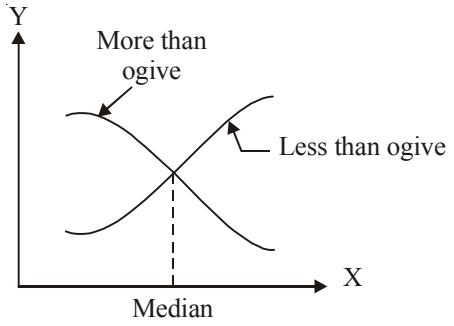
3. x_i = class mark = $\frac{Upper\ Limit + Lower\ Limit}{2}$

4. h = class size = Upper Limit – Lower limit

5. Modal class → A class interval having maximum frequency.

6. Median class → A class interval is which cumulative frequency is greater then and nearest to $\frac{n}{2}$ $\left(n = \Sigma f_i\right)$

7. The median of a group data can be obtained graphically as the x coordinate of the point of intersection of more than and less than ogive.



8. It mean of $x_1, x_2, ... x_n$ is $\overline{x}$ then

(a) Mean of $kx_1, kx_2 ....., kx_n$ is $k\overline{x}$

(b) Mean of $\frac{x_1}{k}, \frac{x_2}{k} ..... \frac{x_n}{k}$ is $\frac{\overline{x}}{k}$

(c) Mean of $x_1 + k, x_2 + k, ..., x_n + k$ is $\overline{x} + k$

(d) Mean of $x_1 - k, x_2 - k, ... x_n - k$ is $\overline{x} - k$

9. It mean of n_1 observation is $\overline{x}_1$ and mean of n_2 observation is $\overline{x}_2$ then their combined

$$\text{Mean} = \frac{n_1 \overline{x}_1 + n_2 \overline{x}_2}{n_1 + n_2}$$

10. $\Sigma xi = n\overline{x}$

11. Range of given data is given by

Highest observation – Lowest observation

12. Graphical Representation of Mode is a Histogram

VERY SHORT ANSWER TYPE(I) QUESTIONS

1. What is the mean of first 12 prime numbers?

2. The mean of 20 numbers is 18. If 2 is added to each number, what is the new mean?

3. The mean of 5 observations 3, 5, 7, x and 11 is 7, find the value of x.

4. What is the median of first 5 natural numbers?

5. What is the value of x, if the median of the following data is 27.5?

24, 25, 26, $x + 2$, $x + 3$, 30, 33, 37

6. What is the mode of the observations 5, 7, 8, 5, 7, 6, 9, 5, 10, 6.

7. The mean and mode of a data are 24 and 12 respectively. Find the median.

8. Write the class mark of the class 19.5 – 29.5.

9. Multiple Choice Question

(i) If the class intervals of a frequency distribution are 1 – 10, 11 – 20, 21 – 30, ..., 51 – 60 then the size of even class is

(a) 9 (b) 10 (c) 11 (d) 5.5

(ii) If the class intervals of a frequency distribution are 1 – 10, 11 – 20, 21 – 30 ...., 61 – 70, Then the upper limit of 21 – 30 is

(a) 21 (b) 30

(c) 30.5 (d) 20.5

(iii) Consider the frequency distribution.

Class	0 – 5	6 – 11	12 – 17	18 – 23	24 – 29
Frequency	13	10	15	8	11

The upper limit of median class is

(a) 17 (b) 17.5 (c) 18 (d) 18.5

(iv) Daily wages of a factory workers are recorded as:

Daily wages in ₹	121 – 126	127 – 132	133– 138	139 – 144	145 – 150
No. of workers	5	27	20	18	12

The lower limit of Modal class is

(a) ₹ 127 (b) ₹ 126 (c) ₹ 126.5 (d) ₹ 133

(v) For the following distribution

Class	0 – 5	5 – 10	10 – 15	15 – 20	20 – 25
Frequency	10	15	12	20	9

The sum of Lower limits of the median class and modal class is

(a) 15 (b) 25 (c) 30 (d) 35

10. Fill in the blank

(a) Mode = 3_____ – 2 _____

(b) An ogive curve is used to determine _____

(c) If the point of intersection of more than and less than ogiven is (20. 5, 30.7) then the median is _____

(d) The mode of a frequency distribution is determined graphically by _____

(e) If the mode is 8 and mean is also 8 then median will be _____

(f) The measure of central tendency which cannot be determined graphically is _____

(g) If the class marks of a continuous frequency distribution are 22, 30, 38, 46, 54, 62 then the class corresponding to class mark 46 is _____

(h) Construction of cumulative frequency distribution table is useful in determining _____

(i) The step deviation formula for finding mean is _____

(j) The formula to find median of grouped data is _____

(k) The formula to find mode of grouped data is _____

(l) The Range of the observations 255, 125, 130, 160, 185, 170, 103 is _____

(m) Class mark is $\frac{1}{2}$ (_____ + _____)

(n) The median of Ist ten prime numbers is _____

(o) The assumed mean method to find mean is _____

SHORT ANSWER TYPE QUESTIONS (I)

11. The mean of 11 observation is 50. If the mean of first Six observations is 49 and that of last six observation is 52, then find sixth observation.

12. Find the mean of following distribution

x	12	16	20	24	28	32
f	5	7	8	5	3	2

13. Find the median of the following distribution

x	10	12	14	16	18	20
f	3	5	6	4	4	3

14. Find the mode of the following frequency distribution.

Class	0–5	5–10	10 –15	15–20	20–25	25–30
Frequency	2	7	18	10	8	5

15. Draw a 'less than' ogive of the following data

Marks	**No. of students**
Less than 20	0
Less than 30	4
Less than 40	16
Less than 50	30
Less than 60	46
Less than 70	66
Less than 80	82
Less than 90	92
Less than 100	100

16. Write the following data into less than cummulative frequency distribution table.

Marks	0–10	10–20	20–30	30–40	40–50
No. of students	7	9	6	8	10

17. Find mode of the following frequency distribution.

Class Interval	25 – 30	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55
Frequency	25	34	50	42	38	14

(CBSE 2018 - 19)

18. What is the median of the following data? (CBSE 2011)

x	10	20	30	40	50
f	2	3	2	3	1

19. Mean of a frequency distribution ($\bar{x}$) is 45. It $\Sigma f_i = 20$ find $\Sigma f_i x_i$

(CBSE 2011)

SHORT ANSWER TYPE QUESTIONS (II)

20. If the mean of the following distribution is 54, find the value of P.

Class	0–20	20–40	40–60	60–80	80–100
Frequency	7	P	10	9	13

21. Find the median of the following frequency distribution.

C.I.	0–10	10–20	20–30	30–40	40–50	50–60
f	5	3	10	6	4	2

22. The median of following frequency distribution is 24 years. Find the missing frequency x.

Age (In years)	0–10	10–20	20–30	30–40	40–50
No. of persons	5	25	x	18	7

23. Find the median of the following data.

Marks	Below 10	Below 20	Below 30	Below 40	below 50	Below 60
No. of student	0	12	20	28	33	40

24. Draw *a* 'more than type' ogive of the following data

Weight (In kg.)	30–35	35–40	40–45	45–50	50–55	55–60
No. of Students	2	4	10	15	6	3

25. Find the mode of the following data.

Height (In cm)	Above 30	Above 40	Above 50	Above 60	Above 70	Above 80
No. of plants	34	30	27	19	8	2

26. The following table represent marks obtained by 100 students in a test.

Marks obtained	30 – 35	35 – 40	40 – 45	45 – 50	50 – 55	55 – 60	60 – 65
No. of students	14	16	28	23	18	8	3

Find mean marks of the students. (CBSE 2018 -19)

27. The following table represent pocket allowance of children of a colony. The mean pocket allowance is ₹ 18. Find missing frequency.

Daily pocket allowance	11 – 13	13 – 15	15 – 17	17 – 19	19 – 21	21 – 23	23 – 25
No. of children	3	6	9	13	k	5	4

(CBSE – 2018)

28. Find mode of the following frequency distribution.

Class Interval	0–20	20–40	40–60	60–80	80–100
No. of Students	15	18	21	29	17

The mean of above distribution is 53. Use Empirical formula to find approximate value of median.

LONG ANSWER TYPE QUESTIONS

29. The mean of the following data is 53, Find the values of f_1 and f_2.

C.I	0–20	20–40	40–60	60–80	80–100	Total
f	15	f_1	21	f_2	17	100

30. If the median of the distribution given below is 28.5, find the values of x and y.

C.I	0–10	10–20	20–30	30–40	40–50	50–60	Total
f	5	8	x	15	y	5	60

31. The median of the following distribution is 35, find the values of a and b.

C.I	0–10	10–20	20–30	30–40	40–50	50–60	60–70	Total
f	10	20	a	40	b	25	15	170

32. Find the mean, median and mode of the following data

C.I	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
f	2	3	6	7	14	12	4	2

33. The rainfall recorded in a city for 60 days is given in the following table.

Raifall (In cm)	0–10	10–20	20–30	30–40	40–50	50–60
No. of Days	16	10	8	15	5	6

Calulate the median rainfall using a more than type ogive.

34. Find the mean of the following distribution by step- deviation method

Daily Expenditure (in ₹)	100–150	150–200	200–250	250–300	300–350
No. of Households	4	5	12	2	2

35. The distribution given below show the marks of 100 students of a class.

Marks	**No. of students**
0–5	4
5–10	6
10–15	10
15–20	10
20–25	25
25–30	22
30–35	18
35–40	5

Draw a less than type and a more than type ogive from the given data. Hence obtain the median marks from the graph.

36. The annual profit earned by 30 factories in an industrial area is given below. Draw both ogives for the data and hence find the median.

Profit (₹ in lakh)	No. of Factories
More than or equal to 5	30
More than or equal to 10	28
More than or equal to 15	16
More than or equal to 20	14
More than or equal to 25	10
More than or equal to 30	7
More than or equal to 35	3
More than or equal to 40	0

37. Convert the following distribution into 'Less than' and then draw its ogive

(CBSE 2018 -19)

Class Interval	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Frequency	7	5	8	10	6	6	8

38. If mean of the given distribution is 65.6 find the mining frequency. (CBSE 2017)

Class Interval	10 – 30	30 – 50	50 – 70	70 – 90	90 – 110	110 – 130	Total
Frequency	5	8	f_1	20	f_2	2	50

ANSWERS AND HINTS

1. 16.4 approx.

2. 20

3. 9

4. 3

5. $x = 25$

6. 5

7. Median = 20

8. 24.5

9. (i) B First make intervals continuous, Then find class size

(ii) C

(iii) C

(iv)C (Make continuous intervals Max. frequency is 27)

(v) B $\begin{bmatrix} \textit{Modal class}\ 15-20 \\ \textit{Median class}\ 10-15 \end{bmatrix}$

10. (a) 3 Median – 2 mean (b) Median

(c) 20.5 (d) Histogram

(e) 8 (f) Mean

(g) 42 – 50 (as difference b/w 2 consecutive observation is 8

$\therefore$ Subtract $\frac{8}{2}$ form 46 for Lower Limit, Add $\frac{8}{2}$ to 46 for upper Limit)

(h) Median (i) $\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

(j) Median $= l + \left(\frac{\frac{n}{2} - Cf_0}{f_1} \right) \times h$ (k) Mode $= l + \frac{(f_1 - f_o)}{(2f_1 - f_o - f_2)} \times h$

(l) Range = 255 – 103 = 152 (m) $\frac{1}{2}$ (upper limit + Lower limit)

(o) $\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$

11. 56 **12.** 20

13. 14.8 **14.** 12.89 approx.

16.

Marks	**No. of students**
less than 10	7
less than 20	16
less than 30	22
less than 40	30
less than 50	40

17.

Class Interval	**Frequency**
25 – 30	25
30 – 35	34 f_0
35 – 40	50 f_1
40 – 45	42 f_2
45 – 50	38
50 – 55	14

$$\text{Mode} = l + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h = 35 + \frac{(50-34)}{(100-34-42)} \times 5 = 35 + \frac{16 \times 5}{24}$$

$= 35 + 3.33 = 38.33$

18.

xi	*fi*	*Cfi*
10	2	2
20	3	5
30	2	7
40	3	10
50	1	11
Total	11	

N = 11 (odd)

$$\text{Median} = \left(\frac{N+1}{2}\right)^{th} \text{ observation} = 6\text{th observation} = 30$$

19. $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} \Rightarrow 45 = \frac{\Sigma f_i x_i}{20} \Rightarrow \Sigma f_i x_i = 900$

20. 11 **21.** 27

22. 10 **23.** 30

25. 63.75 cm

26.

Mark	*xi*	*di*	*ui*	*fi*	*fiui*
30 – 35	32.5	– 15	– 3	14	– 42
35 – 40	37.5	– 10	– 2	16	– 32
40 – 45	42.5	– 5	– 1	28	– 28
45 – 50	47.5 = a	0	0	23	0
50 – 55	52.5	5	1	18	18
55 – 60	57.5	10	2	8	16
60 – 65	62.5	15	3	3	9
				110	–59

$$\bar{x} = a + \frac{\Sigma fiui}{\Sigma fi} \times h = 47.5 - \frac{59}{110} \times 5 = 47.5 - 2.68 = 44.82$$

27. (Make Table just like Q 26)

$$\bar{x} = a + \frac{\Sigma fiui}{\Sigma fi} \times h$$

$$18 = 18 + \frac{(k-8)}{40+k} \times 2$$

$$2k - 16 = 0$$

$$k = 8$$

28. $\text{Mole} = l + \frac{(f_1 - f_0)}{(2f_1 - f_0 - f_2)} \times h$

$$= 60 + \frac{(29-21)}{(2\times 29 - 21 - 17)} \times 20 = 68$$

Mode = 3 Median – 2 mean

68 = 3 Median – 2 × 53

$\frac{68 \times 106}{3}$ = Median

Median = 54

29. $f_1 = 18, f_2 = 29$

30. $x = 20, y = 7$

31. $a = 35, b = 25$

32. Mean = 32.4, median = 33, mode = 34.39 approx.

33. Median = 25 cm

34. Mean = 211

35. Median = 24

36. Median = ₹ 17.5 lakhs.

37.

Less than	*f*	*Cf*
Less than 40	7	7
Less than 50	5	12
Less than 60	8	20
Less than 70	10	30

Less than 80	6	36
Less than 90	6	42
Less than 100	8	50

Plot (40,7), (50, 12), (60, 20), (70, 30) (80, 36), (90, 42), (100, 50)

Join free hand to get ogive.

38.

C.I	**fi**	**xi**	**fixi**
10 – 30	5	20	100
30 – 50	8	40	320
50 – 70	f_1	60	$60f_1$
70 – 90	20	80	1600
90 – 110	f_2	100	$100f_2$
110 – 130	2	120	240
	$\mathbf{35 + f_1 + f_2}$		$\mathbf{2260 + 60\, f_1 + 100\, f_2}$

$35 + f_1 + f_2 = 50 \Rightarrow f_1 + f_2 = 15$ (1)

$$\bar{x} = \frac{\Sigma fixi}{\Sigma fi}$$

$$65.6 = \frac{2260 + 60\, f_1 + 100\, f_2}{50}$$

$$\Rightarrow 3f_1 + 5f_2 = 51 \quad (2)$$

Solve (1) & (2) $f_1 = 12, f_2 = 3$

❐❐❐

PRACTICE-TEST

Statistics

Time : 1 Hr. ***M.M. : 20***

SECTION-A

1. What is the class mark of a class $a - b$. **1**

2. Find the mean of all the even numbers between 11 and 21. **1**

3. An ogive curve is used to detemine **1**

(a) Range (b) Mean (c) Mode (d) Median

4. State True/False **1**

Mean can be determined graphically

SECTION-B

5. The mean of 50 observations is 20. If each observation is multiplied by 3, then what will be the new mean? **2**

6. The mean of 10 observations is 15.3. If two observations 6 and 9 are replaced by 8 and 14 respectively. Find the new mean. **2**

7. Write the modal class for the following frequency distribution **2**

Classes	1 – 4	5 – 8	9 – 12	13 – 16	17 – 20	21 – 24
frequency	8	9	1	12	8	9

SECTION-C

8. Find the mean: **3**

Marks	less than 20	less than 40	less than 60	less than 80	less than 100
No. of Students	4	10	28	36	50

9. Find the value of x if the mode is given to be 58 years. **3**

Age (in years)	20–30	30–40	40–50	50–60	60–70	70–80
No. of patients	5	13	x	20	18	19

SECTION-D

10. The mean of the following frequency distribution is 57.6 and the number of observations is 50. Find the missing frequencies f_1 & f_2. **4**

Class Interval	0–20	20–40	40–60	60–80	80–100	100–120
frequency	7	f_1	12	f_2	8	5

OR

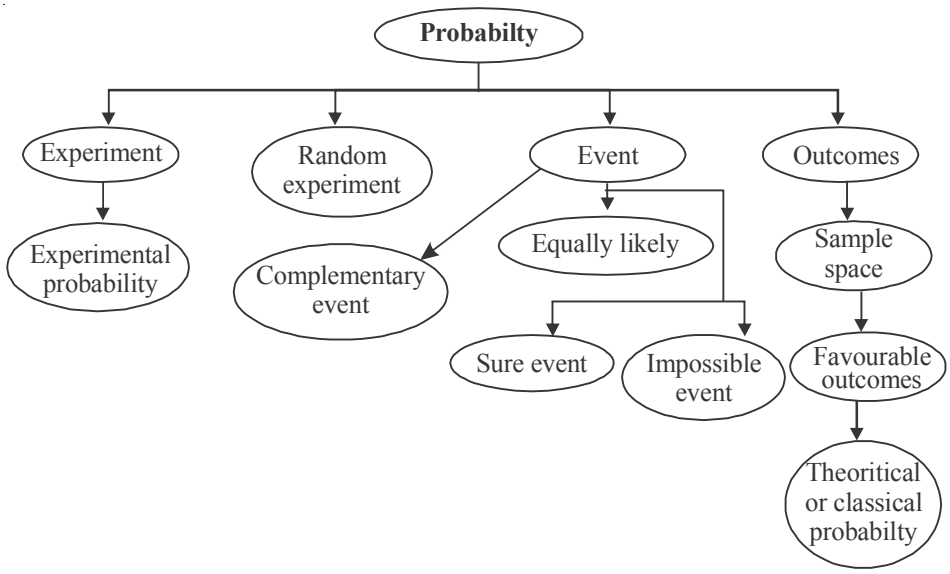
Following is the age distribution of cardiac patients admitted during a month in a hospital:

Age (in years)	20–30	30–40	40–50	50–60	60–70	70–80
No. of patients	2	8	15	12	10	5

Draw a 'less than type' and 'more than type' ogives and from the curves, find the median.

CHAPTER

15 Probability



POINTS TO REMEMBER

1. Probability is a quantitative measure of likelihood of occurrence of an event.
2. Probability of an event $E = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of outcomes}}$
3. $0 \le P(E) \le 1$
4. If $P(E) = 0$ then it is an impossible event.
5. If $P(E) = 1$ then it is sure event.
6. If E is an event than not E($\overline{E}$) is called complementary event.
7. $P(\overline{E}) = 1 - P(E) \Rightarrow P(E) + P(\overline{E}) = 1$
8. Probability of an event is never negative.
9. Sample space : The collection of all possible outcomes of an event.

Examples of Sample space

1. When one coin is tossed then S = H, T
2. When two coins are tossed then S = HH, TT, HT, TH
3. When three coins are tossed than S = HHH, TTT, HTT, THT, TTH, THH, HTH, HHT
4. When four coins are tossed then S = HHHH, TTTT, HTTT, THTT, TTHT, TTTH, HHHT, HHTH, HTHH, THHH, HTHT, THTH, TTHH, HHTT, THHT, HTTH.

1 coin	2coins	3 coins	4 coins
2 outcomes	2 × 2 outcomes	2 × 2 × 2 = 8 outcomes	2 × 2 × 2 × 2 = 16 outcomes

1. When a die is thrown once then S = 1, 2, 3, 4, 5, 6, n(S) = 6
2. When two dice are thrown together or A die is thrown twice then

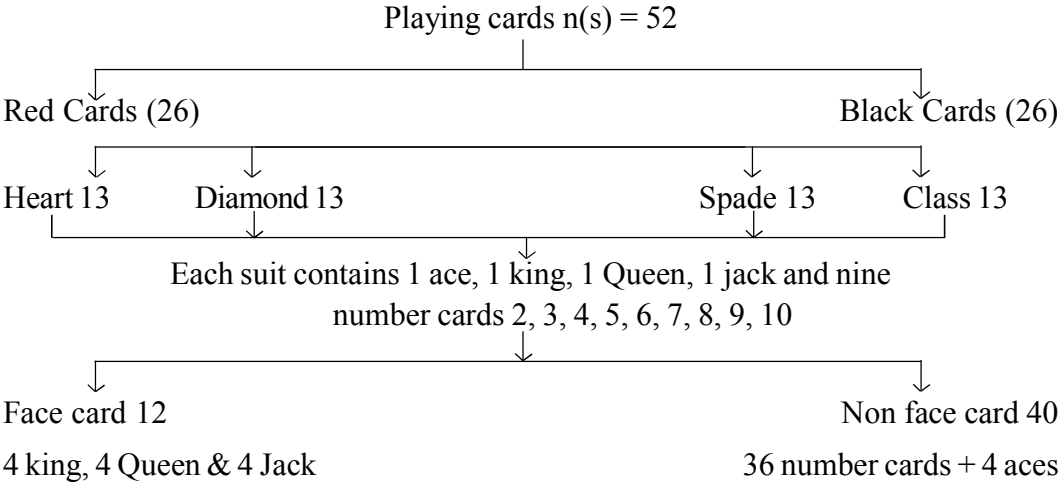
S = (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

n(S) = 6 × 6 = 36

3. When 3 dice are thrown or a die is thrown thrice then

n(S) = 6 × 6 × 6 = 36,

n(S) → no. of outcomes in sample space



VERY SHORT ANSWER TYPE QUESTIONS

1. Fill in the Blanks

(a) The probability of an event is greater than or equal to ..... and is less than or equal to ..... [NCERT]

(b) The probability of an impossible event is .....

(c) The probability of an event that is certain to happen is ..... and such an event is called ..... [NCERT]

(d) The sum of probabilities of all the elementary events of an experiment is ..... [NCERT]

(e) Probability of an event E + probability of the event not E is equal to ..... [NCERT]

(f) If probability of winning a game is 4/9, then the probability of its losing is .....

(g) If coin is tossed twice, then the number of possible outcomes is .....

(h) If a die is thrown twice, then the number of possible outcomes is .....

2. State True/False

(a) The probability of an event can be negative.

(b) The probability of an event is greater than 1.

3. Multiple Choice Questions

(a) Which of the following cannot be the probability of an event? [NCERT]

(A) 0.7 (B) $\frac{2}{3}$ (C) – 1.5 (D) 15%

(b) Which of the following can be the probability of an event?[NCERT Exemplar]

(A) – 0.04 (B) 1.004 (C) $\frac{18}{23}$ (D) $\frac{8}{7}$

(c) An event is very unlikely to happen, its probability is closest to [NCERT Exemplar]

(A) 0.0001 (B) 0.001 (C) 0.01 (D) 0.1

(d) Out of one digit prime numbers, one number is selected at random. The probability of selecting an even number is:

(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{4}{9}$ (D) $\frac{2}{5}$

(e) When a die is thrown, the probability of getting an odd number less than3 is:

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 0

(f) Rashmi has a die whose six faces show the letters as given below

A	B	C	D	A	C

If she throws the die once, then the probability of getting C is

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{1}{6}$

(g) A card is drawn from a well shuffled pack of 52 playing cards. The event E is that the card drawn is not a face card. The number of outcomes favourable to the event E is

(A) 51 (B) 40 (C) 36 (D) 12

4. Choose the correct answer from the given four options

(i) If the probability of an even is 'p' the probability of its complementary event will be:

(A) p – 1 (B) p (C) 1 – p (D) $1-\frac{1}{p}$

(ii) In a family of 3 children, the probability of having atleast one boy is:

[CBSE 2014]

(A) $\frac{7}{8}$ (B) $\frac{1}{8}$ (C) $\frac{5}{8}$ (D) $\frac{3}{4}$

(iii) The probability of a number selected at random from the numbers 1, 2, 3, ... 15 is a multiple of 4 is:

(A) $\frac{4}{15}$ (B) $\frac{2}{15}$ (C) $\frac{1}{5}$ (D) $\frac{1}{3}$

(iv) The probability that a non-leap year selected at random will contains 53 Mondays is:

(A) $\frac{1}{7}$ (B) $\frac{2}{7}$ (C) $\frac{3}{7}$ (D) $\frac{5}{7}$

(v) A bag contains 6 red and 5 blue balls. One ball is drawn at random. The probability that the ball is blue is:

(A) $\frac{2}{11}$ (B) $\frac{5}{6}$ (C) $\frac{5}{11}$ (D) $\frac{6}{11}$

(vi) One alphabet is chosen from the word MATHEMATICS. The probability of getting a vowel is:

(A) $\frac{6}{11}$ (B) $\frac{5}{11}$ (C) $\frac{3}{11}$ (D) $\frac{4}{11}$

5. A card is drawn at random from a pack of 52 playing cards. Find the probability that the card drawn is neither an ace nor a king.

6. Out of 250 bulbs in a box, 35 bulbs are defective. One bulb is taken out at random from the box. Find the probability that the drawn bulb is not defective.

7. Non Occurance of any event is 3:4. What is the probability of Occurance of this event?

8. If 29 is removed from (1, 4, 9, 16, 25, 29) then find the probability of getting a prime number.

9. A card is drawn at random from a deck of playing cards. Find the probability of getting a face card.

10. In 1000 lottery tickets there are 5 prize winning tickets. Find the probability of winning a prize if a person buys one ticket.

11. One card is drawn at random from a pack of cards. Find the probability that it is a black card.

12. A die is thrown once. Find the probability of getting a perfect square.

13. Two dice are rolled simultaneously. Find the probability that the sum of the two numbers appearing on the top is more than and equal to 10.

14. Find the probability of multiples of 7 in 1, 2, 3, ...,33, 34, 35.

SHORT ANSWER TYPE QUESTIONS-I

15. A card is drawn at random from a well shuffled pack of 52 playing cards. Find probability of getting neither a red card nor a queen. [CBSE 2016]

16. Two different dice are rolled together. Find the probability (a) of getting a doublet, (b) of getting a sum of 10, of the numbers on the two dice.

[CBSE 2018]

17. A box contains 12 balls of which some are red in colour. If 6 more red balls are put in the box and a ball is drawn at random, the probability of drawing a red ball doubles than what it was before. Find the number of red balls in the box. [CBSE 2018]

18. An integer is chosen random between 1 and 100. Find the probability that (i) it is divisible by 8, (ii) Not divisible by 8. [CBSE 2018]

19. Three different coins are tossed together. Find the probability of getting (i) exactly two heads, (ii) at least two heads

20. Cards marked with number 3, 4, 5, ... 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box. Find the probability that the selected cards bears a perfect square number. [CBSE 2016]

SHORT ANSWER TYPE QUESTIONS-II

21. A number x is selected at random from the numbers 1, 2, 3 and 4. Another number y is selected at random from the numbers 1, 4, 9 and 16. Find the probability that the product of x and y is less than 16. [CBSE 2016]

22. In a single throw of a pair of different dice, what is the probability of getting (a) a prime number on each dice, (b) a total of 9 or 11. [CBSE 2016]

23. A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag. [CBSE 2017]

24. Two dice are rolled once. Find the probability of getting such numbers on the two dice,

(a) whose produce is 12.

(b) Sum of numbers on the two dice is atmost 5.

25. There are hundred cards in a bag on which numbers from 1 to 100 are written. A card is taken out from the bag at random. Find the probability that the number on the selected card. [CBSE 2016]

(a) It is divisible by 9 and is a perfect square

(b) is a prime number greater than 80.

26. In a lottery, there are 10 prizes and 25 are empty. Find the probability of getting a prize. Also verify $P(E) + P(\overline{E}) = 1$ for this event.

27. P(winning) = $\frac{x}{12}$, P(Losing) = $\frac{1}{3}$. Find x.

LONG ANSWER TYPE QUESTIONS

28. Cards marked with numbers 3, 4, 5, ...,50 are placed in a box and mixed thoroughly. One card is drawn at random from the box, find the probability that the number on the drawn card is

(i) divisible by 7 (ii) a two digit number.

29. A bag contains 5 white balls, 7 red balls, 4 black balls and 2 blue balls. One ball is drawn at random from the bag. Find the probability that the balls drawn is

(i) White or blue (ii) red or black

(iii) not white (iv) neither white nor black

30. The king, queen and jack of diamonds are removed from a pack of 52 playing cards and the pack is well shuffled. A card is drawn from the remaining cards.

Find the probability of getting a card of

(i) diamond (ii) a jack

31. The probability of a defective egg in a lot of 400 eggs is 0.035. Calculate the number of defective eggs in the lot. Also calculate the probability of taking out a non defective egg from the lot.

32. In a fair at a game stall, slips marked with numbers 3,3,5,7,7,7,9,9,9,11 are placed in a box. A person wins if the mean of numbers are written on the slip. What is the probabilty of his losing the game?

33. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

(i) a two digit number (ii) a perfect square number

(iii) a number divisible by 5.

34. A card is drawn at randown from a well shuffled deck of playing cards. Find the probability that the card drawn is

(i) a card of spade or an ace (ii) a red king

(iii) neither a king nor a queen (iv) either a king or a queen

35. A card is drawn from a well shuffled deck of playing cards. Find the probability that the card drawn is

(i) a face card (ii) red colour face card

(iii) black colour face card

36. Ramesh got ₹ 24000 as Bonus. He donated ₹ 5000 to temple. He gave ₹ 12000 to his wife, ₹ 2000 to his servant and gave rest of the amount to his daughter. Calculate the probability of

(i) wife's share (ii) Servant's Share

(iii) daughter's share.

37. 240 students reside in a hostel. Out of which 50% go for the yoga classes early in the morning, 25% go for the Gym club and 15% of them go for the morning walk. Rest of the students have joined the laughing club. What is the probability of students who have joined laughing club?

38. A box contains cards numbered from 11 to 123. A card is drawn at random from the box. Find the probability that the number on the drawn card is:[CBSE 2018]

(i) A square number (ii) a multiple of 7.

39. A die is thrown twice. Find the probability that:

(i) 5 will come up at least once

(ii) 5 will not come up either time [CBSE 2019]

40. Cards marked 1, 3, 5 .... 49 are placed in a box and mixed thoroughly. One card is drawn from the box. Find the probability that the number on the card is : [CBSE 2017]

(i) divisible by 3 (ii) a composite number

(iii) not a perfect square (iv) multiple of 3 and 5

41. Red queens and black jacks are removed from a pack of 52 playing cards. Find the probability that the card drawn from the remaining cards is: [CBSE 2015]

(i) a card of clubs or an ace (ii) a black king

(iii) neither a jack nor a king (iv) either a king or a queen

42. A box contain 100 red cards, 200 yellow cards and 50 blue cards. If a card is drawn at random from the box, find the probability that it will be: [CBSE 2012]

(a) a blue card

(b) not a yellow card

(c) neither yellow nor a blue card

ANSWERS AND HINTS

1. (a) 0 and 1 (b) 0 (c) 1 and sure event (d) 1
(e) 1 (f) 5/9 (g) 4 (h) 36

2. (a) False, because $0 \le P(A) \le 1$

(b) False, because $0 \le P(A) \le 1$

3. (a) (C) (b) (C)
(c) (A) (as unlikely to happen) (d) (B) (prime no. 2, 3, 5, 7)
(e) (A) (f) (A) (probability 2/6)
(g) (B) (Face card 12 Remaining cards 40)

4. (i) (C) ($P + \overline{P} = 1$)

(ii) (A) (Sample space = bbb, bbg, bgb, gbb, ggg, ggb, gbg, ggb)

(iii)(C) (Probability 3/15)

(iv)(A) (Total weeks 52, Remaining day 1, sample space = {S, M, Tu, W, Th, F, Sat})

(v) (C)

(vi) (D) (vowels A, A, E, I)

5. Total = 52

No. of Aces = 4

No. of kings = 4

$$P(\text{neither ace nor king}) = \frac{44}{52} = \frac{11}{13}$$

6. $$P(\text{not defective}) = 1 - \frac{35}{250} = \frac{43}{50}$$

7. Total case 3 + 4 = 7

$$P(\text{occurrence}) = \frac{4}{7}$$

8. P(prime no.) = 0

9. Face card 12

$$P(\text{face card}) = \frac{12}{52} = \frac{3}{13}$$

10. Probability of winning = $\frac{5}{1000} = 0.005$

11. Total black cards 26, $\frac{26}{52} = \frac{1}{2}$

12. Sample space 1, 2, 3, 4, 5, 6
Perfect square 1, 4

P(perfect square) = $\frac{2}{6} = \frac{1}{3}$

13. Favourable cases (4, 6), (5, 5), (6, 4), (5, 6), (6, 5), (6, 6)

P(sum of two numbers is ≥ 10) = $\frac{6}{36} = \frac{1}{6}$

14. Multiples of 7 are 7, 14, 21, 28, 35

Probability (multiple of 7) = $\frac{5}{35} = \frac{1}{7}$

15. No. of red cards = 26
No. of Queens = 04 – 2 = 02 (as 2 red queens are included already)
No. of cards that are neither red nor queen = 56 – (26 + 2) = 24

Required probability = $\frac{24}{52} = \frac{6}{13}$

16. (i) Doublets are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

Required probability = $\frac{6}{36} = \frac{1}{6}$

(ii) Sum 10 cases (4, 6), (5, 5), (6, 4)

Required probability = $\frac{3}{36} = \frac{1}{12}$

17. $\frac{x+6}{18} = 2\left(\frac{x}{12}\right) \Rightarrow x = 3$

18. Total outcomes between 1 and 100 = 98

(i) Nos. divisible by 8 = 8, 16, 24, ..., 96
favourable cases = 12

Required probability = $\frac{12}{98} = \frac{6}{49}$

(ii) Probability (integer is not divisible by 8) = $1 - \frac{6}{49} = \frac{43}{49}$

19. Sample space HHH, TTT, HTT, THT, TTH, THH, HTH, HHT

(i) P(exactly 2 heads) = $\frac{3}{8}$

(ii) P(atleast 2 heads) = $\frac{4}{8} = \frac{1}{2}$ [Favourable cases HHT, HTH, HHT, HHH)

20. Total cards = 50 – 3 + 1 = 48

perfect squares are 4, 9, 16, 25, 36, 49

Required probability = $\frac{6}{48} = \frac{1}{8}$

21. Sample space (1, 1), (1, 4), (1, 9), (1, 16)
(2, 1), (2, 4), (2, 9), (2, 16)
(3, 1), (3, 4), (3, 9), (3, 16)
(4, 1), (4, 4), (4, 9), (4, 16)

Favourable cases $xy < 16$

(1, 1), (1, 4), (1, 9), (2, 1), (2, 4), (3, 1), (3, 4), (4, 1)

Required probability = $\frac{8}{16} = \frac{1}{2}$

22. Total outcomes = 36

(a) Favourable outcomes

(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)

Required probability $\frac{9}{36} = \frac{1}{4}$

(b) Favourable outcomes

(3, 6), (4, 5), (5, 4), (6, 3), (5, 6), (6, 5)

Required probability = $\frac{6}{36} = \frac{1}{6}$

23. $\frac{x}{15+x} = 3 \times \frac{15}{15+x}$, x = 45

No. of black balls = 45

24. (a) S = $\begin{Bmatrix} (1,1)(1,2)(1,3)(1,4)(1,5)(1,6) \\ (2,1)(2,2)(2,3)(2,4)(2,5)(2,6) \\ (3,1)(3,2)(3,3)(3,4)(3,5)(3,6) \\ (4,1)(4,2)(4,3)(4,4)(4,5)(4,6) \\ (5,1)(5,2)(5,3)(5,4)(5,5)(5,6) \end{Bmatrix}$

Favourable outcomes (2, 6), (3, 4), (4, 3), (6, 2)

Required probability = $\frac{4}{36} = \frac{1}{9}$

(b) Favourable outcomes (sum ≤ 5)

= (1, 1), (1, 2), (1, 3) (1, 4) (2, 1) (2, 2) (2, 3) (3, 1) (3, 2) (4, 1)

Required probability = $\frac{10}{36} = \frac{5}{18}$

25. (i) Total number = 100

Number divisible by 9 and a perfect square = 9, 36, 81

Required probability $\frac{3}{100}$ = 0.03

(ii) Prime no. > 80 are 83, 89, 97

Required probability $\frac{3}{100}$ = 0.03

26. Total tickets = 35

P(E) = P(getting a prize) = $\frac{10}{35} = \frac{2}{7}$

P(E) = P(not getting a prize) = $\frac{25}{35} = \frac{5}{7}$

P(E) + P($\bar{E}$) = $\frac{2}{7} + \frac{5}{7} = \frac{7}{7} = 1$

27. P(winning) + P(losing) = 1

$\frac{x}{12} + \frac{1}{3} = 1$, x = 8

28. Total cards = 50 – 3 + 1 = 48

(i) No. divisible by 7 are 7, 14, 21, 28, 35, 42, 49

Required probability = $\frac{7}{48}$

(ii) Two digit no. are 10, 11, 12, ... 50

No. of favourable outcomes = 50 – 10 + 1 = 41

Required probability = $\frac{41}{48}$

29. (i) $\frac{5+2}{18}=\frac{7}{18}$ (ii) $\frac{7+4}{18}=\frac{11}{18}$

(iii) $\frac{7+4+2}{18}=\frac{13}{18}$ (iv) $\frac{7+2}{18}=\frac{9}{18}=\frac{1}{2}$

30. (i) Remaining cards = 52 – 3 = 49

Remaining diamonds = 13 – 3 = 10

Required probability = $\frac{10}{49}$

(ii) P(jack) = $\frac{3}{49}$ (as 1 jack has been removed)

31. Total eggs = 400

P(defective eggs) = 0.035

Let defective eggs = x

$$\frac{x}{400} = 0.035$$

$$x = 400 \times 0.035$$

$$x = 14$$

P(non defective) = 1 – 0.035 = 0.965

32. Mean = $\frac{3+3+5+7+7+7+9+9+9+11}{10}=\frac{70}{10}=7$

$$\text{P(he loses)} = 1-\frac{7}{10}=\frac{3}{10}$$

33. Total no. = 90

(i) Two digit no.s 10, 11, 12, ..., 90

No. of favourable cases = 90 – 10 + 1 = 81

Required probability = $\frac{81}{90}=\frac{9}{10}$

(ii) Perfect square no. = 1, 4, 9, 16, 25, 36, 49, 64, 81

Required probability = $\frac{9}{90} = \frac{1}{10}$

(iv) No.s divisible by 5

5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90

Required probability = $\frac{18}{90} = \frac{1}{5}$

34. (i) P(a card of spade or an ace) = $\frac{13+3}{52} = \frac{16}{52} = \frac{4}{13}$

(ii) P(red king) = $\frac{2}{52} = \frac{1}{26}$

(iii)P(neither a king nor a queen) = $1 - \frac{8}{52} = 1 - \frac{2}{13} = \frac{11}{13}$

(iv)P(either a king or a queen) = $\frac{8}{52} = \frac{2}{13}$

35. (i) $\frac{12}{52} = \frac{3}{13}$ (ii) $\frac{6}{52} = \frac{3}{26}$ (iii) $\frac{6}{52} = \frac{3}{26}$

36. (i) P(wifes share) = $\frac{12000}{24000} = \frac{1}{2}$

(ii) P(servant's share) = $\frac{2000}{24000} = \frac{1}{12}$

(iii)P(Daughter's share) = $\frac{5000}{24000} = \frac{5}{24}$

37. 10% students joined laughing club

P(students who have joined laughing clubs) = $\frac{10}{100} = \frac{1}{10}$

38. Total cards = 123 – 11 + 1 = 113

(i) Square numbers 16, 25, 36, 49, 64, 81, 100, 121

Required probability = $\frac{8}{113}$

(ii) Multiple if 7 are 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, 112, 119. Required Probality = $\frac{16}{113}$

39. Total outcomes = 36

(i) P(5 will come up at least once) = $\frac{11}{36}$

Favourable cases (1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6)

(ii) P(5 will not come up either time) = $1-\frac{11}{36}=\frac{25}{36}$

40. S = 1, 3, 5, ..., 49. Total outcome = 25

(i) No. divisible by 3 are 3, 9, 15, 21, 27, 33, 39, 45

Required probability = $\frac{8}{25}$

(ii) Composite No.s 9, 15, 21, 25, 27, 33, 35, 39, 45, 49

Required probability = $\frac{10}{25}=\frac{2}{5}$

(iii)P(not a perfect square) = 1 – P(perfect square) {Perfect square 1, 9, 25, 49}

$= 1-\frac{4}{25} = \frac{21}{25}$

(iv) Multiple of 3 and 5

$\Rightarrow$ Multiple of 15 = 15, 45

Required probability = $\frac{2}{25}$

41. (i) $\frac{16}{52}=\frac{4}{13}$ (ii) $\frac{2}{52}=\frac{1}{26}$

(iii) $1-\frac{8}{52}=1-\frac{2}{13}=\frac{11}{13}$ (iv) $\frac{8}{52}=\frac{2}{13}$

42. (a) P(blue card) = $\frac{50}{350}=\frac{1}{7}$ (b) P(not yellow card) = $\frac{150}{350}=\frac{3}{7}$

(c) P(neither yellow nor blue) = $\frac{100}{350}=\frac{2}{7}$

❐❐❐

PRACTICE-TEST

Probabiltiy

Time : 1 Hr. ***M.M. : 20***

SECTION-A

1. A die is thrown once. find the probability of getting an odd number. 1

2. A bag contains 4 red and 6 black balls. one ball is drawn from the bag at random. Find the probability of getting a black ball. 1

3. A single letter is selected from the word PROBABILITY. The probability it is a vowel = ... 1

4. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. The number of rotten apples are ..... (CBSE 2017)1

SECTION-B

5. Find the probability of having 53 friday in a year. 2

6. One card is drawn at random from the well shuffled pack of 52 cards. Find the probability of getting a black face card or a red face card. 2

7. A coin is tossed twice. Find the probability of getting atleast one tail. (CBSE 2014) 2

SECTION-C

8. A box contains 5 Red, 4 green and 7 white marbles. One marbles is drawn at random from the box. What is the probability that marble is

   (i) not white (ii) neither red nor white 3

8. A die is thrown once. find the probability that the number.

   (i) is an even prime number (ii) is a perfect square 3

SECTION-D

10. A box contains cards numbered 1,3,5,.....,35. Find the probability that the card drawn is

    (i) a prime number less than 15 (ii) divisible by both 3 and 15 4

OR

From a deck of 52 playing cards, king, queen and jack of a club are removed and a card is drawn from the remaining cards. Find the probability that the card drawn is

(i) A spade (ii) a queen

(iii) A club

□□□

PRACTICE TEST-I (With Solutions)
Class : X
Mathematics (Basic)

Time : 3 hours **Max. Marks : 80**

General Instructions:

1. All the questions are compulsory.
2. The question paper consists of 40 questions and it is divided into four sections A, B, C and D.
3. Section A comprises of 20 questions carrying 1 mark each. Section B comprises of 6 questions carrying 2 marks each. Section C comprises of 8 questions carrying 3 marks each. Section D comprises of 6 questions carrying 4 marks each.
4. There is no overall choice.
5. Use of calculator is not permitted.

SECTION A

1. If p and q are co-prime, then p^2 and q^2 are ..... .

2. If $\Delta ABC \sim \Delta DEF$, BC = 3 cm, EF = 4 cm and ar(ΔABC) = 54 cm^2, then ar(ΔDEF) = ...

3. If $5 \tan \theta - 4 = 0$, then the value of $\frac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta}$ is

(A) $\frac{5}{3}$ (B) $\frac{5}{6}$ (C) 0 (D) $\frac{1}{6}$

4. A die is thrown once. The probability of getting a prime number is:

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{1}{6}$

5. If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then

(A) $k < 4$ (B) $k > 4$ (C) $k \geq 4$ (D) $k \leq 4$

6. If the circumference and the area of a circle are numerically equal, then diameter of the circle is

(A) $\frac{\pi}{2}$ units (B) 2π units (C) 2 units (D) 4 units

7. The next term of the A.P. : $\sqrt{7}, \sqrt{28}, \sqrt{63}$...

(A) $\sqrt{70}$ (B) $\sqrt{84}$ (C) $\sqrt{97}$ (D) $\sqrt{112}$

8. The distance between the points $(a \cos \theta + b \sin \theta, 0)$ and $(0, a \sin \theta - b \cos \theta)$ is:

(A) $a^2 + b^2$ (B) $a + b$ (C) $a^2 - b^2$ (D) $\sqrt{a^2 + b^2}$

9. If a quadratic polynomial f(x) is a square of a linear polynomial, then its zeros are equal. (True/False)

10. From a point lying on the circle, infinite number of tangents can be drawn. (True/False)

11. For what value of p, (– 4) is a zero of the polynomial $x^2 - 2x - (7p + 3)$?

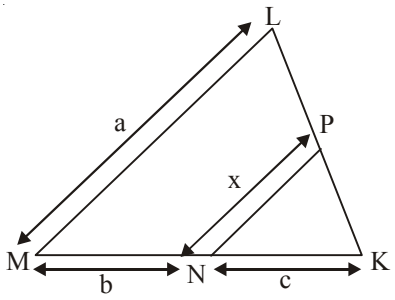
12. Find the number of solutions of the following pair of linear equations:

$$x + 2y - 8 = 0$$
$$2x + 4y = 16$$

13. Find the area of a triangle with vertices (0, 4), (0, 2) and (3, 0)

14. If A(1, 2), B(4, 3) and C(0, 0) are three vertices of parallelogram ABCD, find the coordinates of D.

15. In figure, PN || LM. Express x in terms of a, b and c, where a, b and c are lengths of LM, MN and NK respectively.



16. State Basic Proportionality Theorem.

17. What is the probability that a non-leap year has 53 Mondays?

18. If the total surface area of a solid hemisphere is 462 cm^2, find its diameter.

19. A pole casts a shadow of length $2\sqrt{3}$ m on the ground, when the sun's elevation is 60°, find the height of the pole.

20. If E be an event such that $P(E) = \frac{3}{7}$, what is P(not E) equal to?

SECTION B

21. Given that $\sqrt{2}$ is irrational, prove that $(5 + 3\sqrt{2})$ is an irrational number.

22. For what value of 'k' the system of equation $kx + 3y = 1$, $12x + ky = 2$ has no solution.

23. The length of minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

24. Two cubes each of volume 27 cm^3 are joined end to end to form a solid cuboid. Find the surface area of the resulting cuboid.

25. The following distribution table shows the marks scored by 140 students in an examination:

Marks	0–10	10–20	20–30	30–40	40–50
Number of students	20	45	80	55	40

Calculate the mode of the distribution.

26. An integer is chosen at random between 1 and 100. Find the probability that it is:

(i) divisible by 8. (ii) not divisible by 8.

SECTION C

27. Find the HCF of 180, 252 and 324 by prime factorization method.

28. Find all zeros of the polynomial $2x^4 - 9x^3 + 5x^2 + 3x - 1$, if two of its zeros are $(2+\sqrt{3})$ and $(2-\sqrt{3})$.

29. Solve for x : $\frac{1}{(x-1)(x-2)} + \frac{1}{(x-2)(x-3)} = \frac{2}{3}$; $x \neq 1, 2, 3$

30. The ninth term of an A.P. is equal to seven times the second term and twelth term exceeds five times the third term by 2. Find the first term and the common difference.

31. Prove that in a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

32. Two tangents TP and TQ are drawn to a circle with centre O, from an external point T. Prove that $\angle PTQ = 2\angle QPQ$.

33. Prove that $(\cot\theta - \text{cosec}\,\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

34. In ΔABC, $\angle B = 90°$, BC = 5 cm and AC – AB = 1 cm. Find the value of sin C and cos C.

SECTION D

35. Draw the graph of the following equations and answer the following questions :

$x + y = 5$, $x - y = 5$

(i) Find the solution of the equations from the graph.

(ii) Shade the triangular region formed by the lines and the y-axis.

36. If A and B are $(-2, -2)$ and $(2, -4)$ respectively, find the coordinates of P such that $AP = \frac{3}{7}$ AB and P lies on the line segment AB.

37. Construct ΔABC with BC = 7 cm, $\angle B = 60°$ and AB = 6 cm. Construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of ΔABC.

38. As observed from the top of 100 m high light house from the sea level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light house, find the distance between the two ships. (Use $\sqrt{3}$ = 1.732)

39. A hollow sphere of internal and external diameter 4 cm and 8 cm respectively is melted to form a cone of base diameter 8 cm. Find the height and the slant height of cone.

40. Find the mean and median of the following distribution:

Class	11–13	13–15	15–17	17–19	19–21	21–23	23–25
Frequency	3	6	9	13	18	5	4

ANSWERS AND HINTS

1. Co-prime

2. 96 cm^2

3. (C) 0

4. (C) 1/2

5. (A) $k < 4$

6. (D) 4 units

7. (D) $\sqrt{112}$

8. (D) $\sqrt{a^2 + b^2}$

9. True

10. False

11. $(-4)^2 - 2(-4) - (7p + 3) = 0$, $p = 3$

12. As $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ so, given pair of linear equations has infinitely many solutions.

13. $\Delta = \frac{1}{2}[0(2-0) + 0(0-4) + 3(4-0)] = 6$ square units

14. Let coordinates of D be (x, y).

Coordinates of the mid-point of AC = Coordinates of the mid-point of BD

$$\left(\frac{1+0}{2}, \frac{2+0}{2}\right) = \left(\frac{4+x}{2}, \frac{3+y}{2}\right)$$

$\therefore \quad x = -3$ and $y = -1$

Hence, coordinate of D is $(-3, -1)$

15. $\Delta KLM \sim \Delta KPN$ (AA similarity criterion)

$$\frac{LM}{PN} = \frac{KM}{KN} \Rightarrow \frac{a}{x} = \frac{b+c}{c} \Rightarrow x = \frac{ac}{b+c}$$

16. Correct statement.

17. P(53 Mondays) $= \frac{1}{7}$

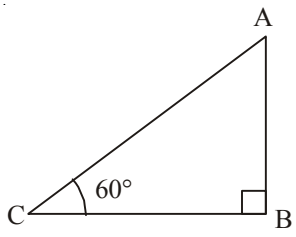
18. $3\pi r^2 = 462 \qquad \Rightarrow r = 7$ cm

Diameter = 14 cm

19. AB be the pole and BC be its shadow.

In ΔABC, $\frac{AB}{BC} = \tan 60°$

$AB = 2\sqrt{3} \times \sqrt{3} = 6$ m



20. P(not E) $= 1 - \frac{3}{7} = \frac{4}{7}$

21. Let us assume that $5 + 3\sqrt{2}$ be a rational number such that it can be written as $5 + 3\sqrt{2} = \frac{a}{b}$; $b \neq 0$, a and b are co-prime numbers.

$$\sqrt{2} = \frac{a-5b}{3b}$$

RHS is rational. So, LHS is also rational which contradict that $\sqrt{2}$ is irrational.

So, our assumption is wrong. Therefore, $5 + 3\sqrt{2}$ is irrational number.

22. For no solution,

$$\frac{k}{12} = \frac{3}{k} \neq \frac{1}{2}$$

$k = \pm 6$ or $k \neq 6$

$\therefore$ $k = -6$

23. Angle swept in 5 minutes = 30°

Area swept in 5 minutes = $\frac{30°}{360°} \times \frac{22}{7} \times 14 \times 14 = 51\frac{1}{3} cm^2$

24. Side of cube = $\sqrt[3]{27} = 3cm$

Length, breadth and height of cuboid is 3 + 3 = 6 cm, 3 cm, 3 cm respectively.

Surface Area of cuboid = $2(6 \times 3 + 3 \times 3 + 3 \times 6) = 90\ cm^2$

25. Modal class = 20 – 30

Mode = $20 + \frac{40-24}{2\times 40-24-36} \times 10 = 28$

26. Number of integers between 1 to 100 is 98.

(i) P(divisible by 8) = $\frac{12}{98} = \frac{6}{49}$

(ii) P(not divisible by 8) = $1 - \frac{6}{49} = \frac{43}{49}$

27. $180 = 2^2 \times 3^2 \times 5$

$252 = 2^2 \times 3^2 \times 7$

$324 = 2^2 \times 3^4$

HCF (180, 252, 324) = $2^2 \times 3^2 = 36$

28. Let $p(x) = 2x^4 - 9x^3 + 5x^2 + 3x - 1$

$(2 + \sqrt{3})$ and $(2 - \sqrt{3})$ are zeros of p(x).

$x - (2 + \sqrt{3})$ and $x - (2 - \sqrt{3})$ are factors of p(x).

$\therefore$ $[x - (2 + \sqrt{3})][x - (2 - \sqrt{3})] = x^2 - 4x + 1$ is also factor of p(x)

By long division,

$p(x) = (x^2 - 4x + 1)(2x^2 - x - 1)$

$= [x - (2 + \sqrt{3})][x - (2 - \sqrt{3})](2x + 1)(x - 1)$

$\therefore$ zeros of p(x) are 1, $\frac{-1}{2}$, $(2 + \sqrt{3})$ and $(2 - \sqrt{3})$

29. $\frac{1}{x-2}\left[\frac{1}{x-1} + \frac{1}{x-3}\right] = \frac{2}{3}$

$\Rightarrow \quad \frac{1}{x-2} \times \frac{2(x-2)}{(x-1)(x-3)} = \frac{2}{3}$

$\Rightarrow \quad x^2 - 4x + 3 = 3$

$\Rightarrow \quad x^2 - 4x = 0$

$\Rightarrow \quad x(x - 4) = 0$

either $x = 0$ or $x = 4$

30. $a_9 = 7a_2$

$a + 8d = 7(a + d)$

$\Rightarrow \quad d = 6a$ ...(1)

$a_{12} = 5a_3 + 2$

$\Rightarrow \quad 4a - d + 2 = 0$ ...(2)

from (1) and (2), we have

$a = 1$ and $d = 6$

31. Correct proof

32. Join OP, OQ and PQ.

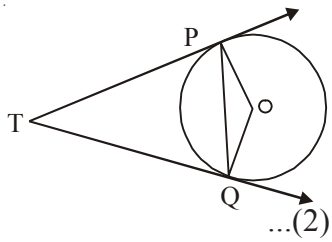
Let $\angle PTQ = x$

In ΔPTQ, $\angle TQP + \angle TPQ + \angle PTQ = 180°$

$\Rightarrow \quad \angle TQP + \angle TPQ = 180° - x$ ...(1)

Also, TP = TQ ($\because$ tangents from an external point)

$\therefore \quad \angle TQP = \angle TPQ$ ...(2)



From (1) and (2),

$$\angle TPQ = 90° - \frac{x}{2} \qquad ...(3)$$

$$\angle OPT = 90°$$

$$\Rightarrow \quad \angle OPQ + \angle TPQ = 90°$$

$$\angle OPQ = \frac{x}{2} \text{ (from 3)}$$

$$2\angle OPQ = \angle PTQ$$

33. LHS = $(\cot\theta - \text{cosec}\,\theta)^2 = \left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)^2$

$$= \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta} = \text{RHS}$$

34. Given, AC – AB = 1...(1)

In ΔABC, $AC^2 = AB^2 + BC^2$

$AC^2 - AB^2 = (5)^2$

(AC – AB) (AC + AB) =25

AC + AB = 25 ...(2)

From (1) and (2), AC = 13 cm and AB = 12 cm

$$\sin C = \frac{12}{13}$$

$$\cos C = \frac{5}{13}$$

35. Correct graph and solution is (5, 0). Shade the required region.

36. D $\leftrightarrow$ (4, 0)

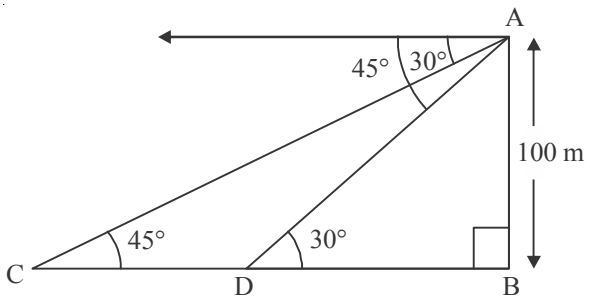
$$\text{ar}(\Delta ABD) = \frac{1}{2}\,[4(-2-0) + 3(0+6) + 4\,(-6+2)] = 3 \text{ square units}$$

$$\text{ar}(\Delta ACD) = \frac{1}{2}\,[4(2-0) + 5(0+6) + 4\,(-6-2)] = 3 \text{ square units}$$

$\therefore$ $\text{ar}(\Delta ABD) = \text{ar}(\Delta ACD)$

37. Correct construction

38.



In ΔABD, $\frac{AB}{BD} = \tan 45° \Rightarrow BD = 100\text{ m}$

In ΔABC, $\frac{AB}{BC} = \tan 30° \Rightarrow \frac{100}{CD+100} = \frac{1}{\sqrt{3}}$

$\Rightarrow$ $CD = 100(1.732 - 1) = 73.2\text{m}$

39. Let height of cone be h cm.

Volume of cone = Volume of hollow sphere

$$\frac{1}{3}\pi(4)^2 h = \frac{4}{3}\pi(4^3 - 2^3)$$

$$l = 14\text{ cm}$$

$$l = \sqrt{(4)^2 + (14)^2} = 2\sqrt{53}\text{ cm}$$

40.

Class	$\mathbf{f_i}$	$\mathbf{x_i}$	$\mathbf{f_i x_i}$	**c.f.**
11–13	3	12	36	3
13–15	6	14	84	9
15–17	9	16	144	18
17–19	13	18	234	31
19–21	18	20	360	49
21–23	5	22	110	54
23–25	4	24	96	58
Total	**58**		**1064**	

Mean $= \frac{1064}{58} = 18$, Median $= 17 + \frac{(29-18)}{13} \times 2 = 18.69$ approx.

PRACTICE TEST-II
Class : X
Mathematics (Basic)

Time : 3 hours **Max. Marks : 80**

General Instructions:

1. All the questions are compulsory.

2. The question paper consists of 40 questions and it is divided into four sections A, B, C and D.

3. Section A comprises of 20 questions carrying 1 mark each. Section B comprises of 6 questions carrying 2 marks each. Section C comprises of 8 questions carrying 3 marks each. Section D comprises of 6 questions carrying 4 marks each.

4. There is no overall choice.

5. Use of calculator is not permitted.

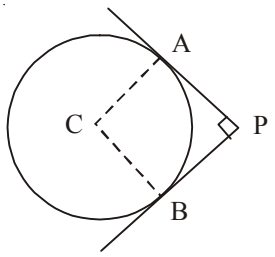
SECTION A

Question number 1 to 20 carry 1 mark each.

1. Find the LCM of 96 and 360 by using Fundamental Theorem of Arithmetic.

2. A line segment is of length 5 cm. If the coordinates of its one end are (2, 2) and that of the other end are (– 1, x), then find the value of x.

3. In figure, PA and PB are two tangents drawn from an external point P to a circle with centre C and radius 4 cm. If PA ⊥ PB, then find the length of each tangents.



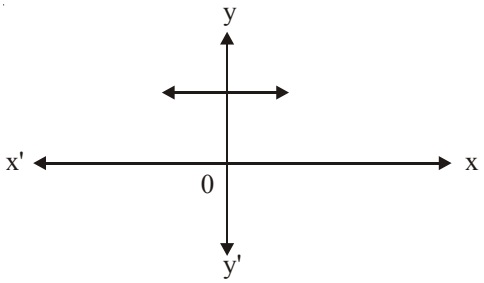
4. The first three terms of an A.P. respectively are 3y – 1, 3y + 5 and 5y + 1. Find the value of y.

5. A die is thrown once. What is the probability that it shows a number greater than 4?

6. A solid sphere of radius r is melted and cast into the shape of a solid cone of height *r*. Find the radius of the base of cone.

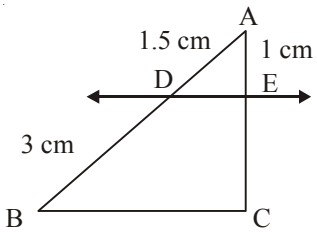
7. The graph of y = p(x) is given in the figure. The number of zeros of p(x) are:

(a) one (b) three (c) zero (d) two



8. In the figure : DE || BC then the value of EC is:

(a) 1 cm (b) 2 cm (c) 1.5 cm (d) 3 cm



9. From a point Q the length of tangent to a circle is 24 cm and distance of Q from the centre is 25 cm. The radius of the circle is:

(a) 7 cm (b) 12 cm (c) 15 cm (d) 24.5 cm

10. The angle of elevation of the top of a 15 metres high tower from a point 15 metres away from its foot is:

(a) 30° (b) 45° (c) 60° (d) 90°

11. The difference between the circumference and the diameter of a circle is 30 cm then the radius of the circle is:

(a) 5 cm (b) 7.7 cm (c) 7 cm (d) 6 cm

12. Probability of event E + Probability of event 'not E' = ...

13. A polynomial of degree two is called ..... polynomial.

14. The line x – y = 8 intersect y-axis at (0, – 8) (T/F)

15. Number of solution in the given pair of equation is infinitely many solutions. (T/F)

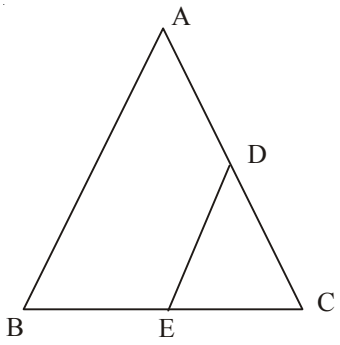
x + 2y – 8 = 0

2x + 4y = 16

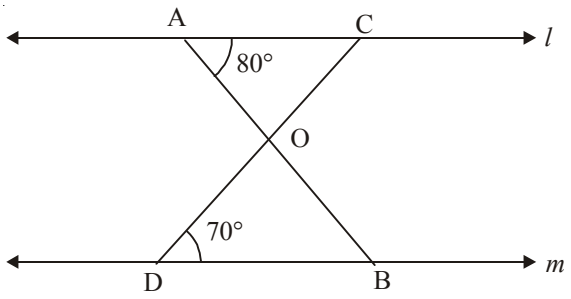
16. $3\cot^2 60° + \sec^2 45 =$ .....

17. Cards marked with numbers 3, 4, 5 .... 50 are placed in a box and mixed thoroughly. A card is drawn at random from the box, find the probability that the selected card bears a perfect square number.

18. In the figure ΔABC, $DE \| AB$. If $AD = 2x$, $DC = x + 3$, $BE = 2x - 1$ and $CE = x$ then find the value of x.



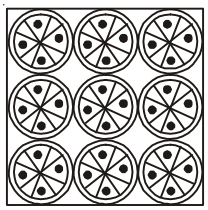
19. In the figure, $l \| m$, $\angle OAC = 80°$, $\angle ODB = 70°$. Is $\Delta OCA \sim \Delta ODB$?



20. Find the value of k, for which one root of the quadratic equation $Kx^2 - 14x + 8 = 0$ is six times the other.

Question number 21 to 26 carry 2 mark each.

21. On a square handkerchief, nine circular designs each of radius 7 cm are made. Find the area of the remaining portion of the handkerchief.



22. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

23. For what value of k, will the following system of equations have no solutions?

$$(3k + 1)x + 3y = 2$$
$$(k^2 + 1)x + (k - 2)y = 5$$

24. A cylindrical tub, whose diameter is 12 cm and height 15 cm is full of ice-cream. The whole ice cream is to be divided into 10 children in equal ice-cream cones, with conical base surmounted by hemispherical top. If the height of conical portion is twice the diameter of base. Find the diameter of conical part of ice-cream cone.

25. Find the mean of the following frequency distribution:

Class	0–5	5–10	10–15	15–20	20–25	25–30
Frequency	1	2	2	6	7	2

26. Cards are marked with the numbers from 2 to 151 are placed in a box and mixed thoroughly. One card is drawn at random from this box. Find the probability that the number on the card is:

(i) a prime number less than 75.

(ii) an odd number.

SECTION C

Question number 27 to 34 carry 3 mark each.

27. Evaluate : $(\cos^2 20° + \cos^2 70°) + \dfrac{\cot 25°}{\tan 65°} + \cot 5° \cot 10° \cot 60° \cot 80° \cot 85°$.

28. QT and RS are medians of a triangle PQR right angled at P. Prove that $4(QT^2 + RS^2) = 5QR^2$.

29. If α and β are zeroes of the polynomial $p(x) = 2x^2 + 11x + 5$, find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta} - 2\alpha\beta$.

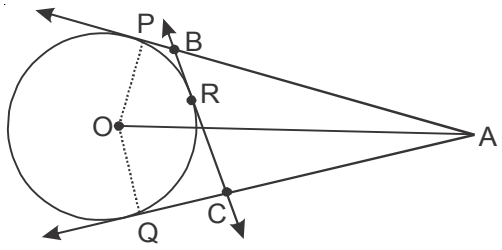
30. Prove that : $\dfrac{\sin\theta}{1-\cos\theta} + \dfrac{\tan\theta}{1+\cos\theta} = \cos\theta \operatorname{cosec}\theta + \cot\theta$.

31. Find the roots of the equation $\dfrac{1}{x+4} + \dfrac{1}{x-7} = \dfrac{11}{30}$ $x \neq -4, 7$.

32. Show that one and only one out of n, n + 2, n + 4 is divisible by 3, where 'n' is any positive integer.

33. The sum of first six terms of an A.P. is 42. The ratio of its 10th term to 30^{th} term is 1 : 3. Calculate the first and 13^{th} term of the A.P.

34. In figure, AB is a chord of a circle, with centre O, such that AB = 16 cm and radius of circle is 10 cm. Tangents at A and B intersect each other at P. Find the length of PA.



35. Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

36. Determine the ratio in which the line $3x + y - 9 = 0$ divides the line-segment joining the points (1, 3) and (2, 7).

37. The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.

38. Due to sudden floods, some welfare "associations jointly requested the government to get 100 tents fixed immediately and offered to contribute 50% of the cost. If the lower part of each tent is of the form of a cylinder of diameter 4.2 m and height 4 m with the conical upper part of same diameter but of height 2.8 m, and the canvas to be used costs ₹ 100 per sq. m, find the amount, the associations will have to pay. $\left[\text{Use } \pi = \frac{22}{7}\right]$

39. The following distribution gives the daily income of 50 workers of a factory:

Daily income (in ₹)	200–250	250–300	300–350	350–400	400–450	450–500
Number of workers	10	5	11	8	6	10

Convert the distribution to a less than type cumulative frequency distribution and draw its ogive. Hence obtain the median daily income.

40. Draw a circle of radius 5 cm. From a point P, 8 cm away from its centre, construct a pair of tangents to the circle. Measure the length of each one of the tangents.

ANSWERS

1. 1440 **2.** x = 6 or x = 2 **3.** PA = PB = 4 cm **4.** y = 5

5. $\frac{1}{3}$ **6.** 2r **7.** No **8.** 2 cm

9. 7 cm **10.** 45° **11.** 7 cm **12.** 1

13. Quadratic **14.** True **15.** True **16.** 3

17. $\frac{1}{8}$ **18.** $\frac{3}{5}$ **19.** Yes **20.** K = 3

21. Area of the remaining portion = 378 cm^2

22. 1.5 is rational number lying between $\sqrt{2}$ and $\sqrt{3}$

23. (–1) **24.** – 6 cm **25.** Mean = 18

26. (i) $\frac{7}{50}$ (ii) $\frac{1}{2}$ (iii) $\frac{11}{150}$

27. $\frac{6+\sqrt{3}}{3}$ **29.** $-\frac{36}{5}$

31. x = 1, 2 **33.** First term = 2, $a_{13} = -26$

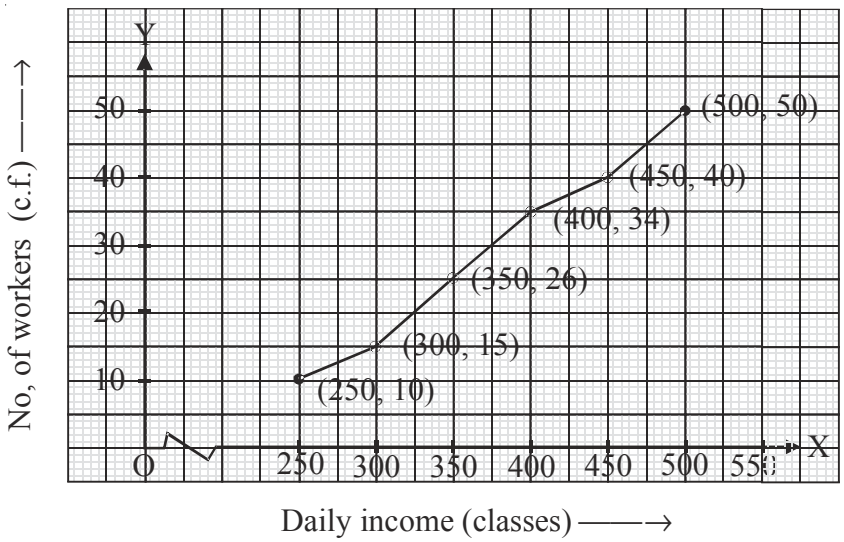
34. $\frac{40}{3}$ cm

35. Speed of the two cars are 60 km/h and 40 km/h respectively

36. Ratio is 3 : 4 internally **37.** Height of the building is 20 metres

38. The associations will have to pay the amount = ₹ 379500

39. Median daily income = ₹ 345



Daily income (classes)	**No. of workers (*c.f.*)**
less than 250	10
less than 300	15
less than 350	26
less than 400	34
less than 450	40
less than 500	50

40. Length of each tangent = $\sqrt{39}$ cm

PRACTICE PAPER- I (WITH SOLUTIONS)
CLASS: X
Mathematics (Standard)

Time : 3 hours **Max. Marks : 80**

General Instructions:

1. All the questions are compulsory.
2. The question paper consists of 40 questions and it is divided into four sections A, B, C and D.
3. Section A comprises of 20 questions carrying 1 mark each. Section B comprises of 6 questions carrying 2 marks each. Section C comprises of 8 questions carrying 3 marks each. Section D comprises of 6 questions carrying 4 marks each.
4. There is no overall choice.
5. Use of calculator is not permitted.

SECTION A

Question number 1 to 20 carry 1 mark each.

1. The LCM of two numbers is 1200. Which of the following cannot be their HCF?

(a) 4 (b) 5 (c) 6 (d) 3

2. The median of a given frequency distribution is found graphically with the help of

(a) histogram (b) frequency curve (c) frequency polygon (d) ogive

3. If the arithmetic mean of x, x + 3, x + 6, x + 9 and x + 12 is 10, then x = ...

(a) 1 (b) 2 (c) 6 (d) 4

4. Two different dice are tossed together. The probability that the product of two numbers on the top of dice is 6 is

(a) $\frac{1}{3}$ (b) $\frac{1}{6}$ (c) $\frac{1}{9}$ (d) $\frac{2}{3}$

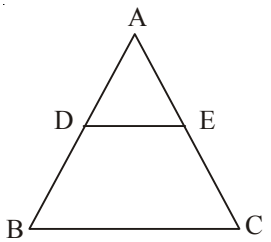
5. A cylinder, a cone and a hemisphere are of same base and have same height. The ratio of their volumes is

(a) 3 : 1 : 2 (b) 1 : 2 : 3 (c) 3 : 2 : 1 (d) 1 : 3 : 2

6. Two isosceles triangles have equal angles and their areas are in the ratio 16 : 25. The ratio of their corresponding heights is

(a) 4 : 5 (b) 5 : 4 (c) 3 : 2 (d) 5 : 7

7. In figure, DE||BC and AD = $\frac{1}{2}$ BD. If BC = 4.5 cm, find DE.



8. If radii of two concentric circles are 4 cm and 5 cm find the length of each chord of one circle which is tangent to the other circle.

9. If the diameter of a circle is increased by 40%, find by how much percentage its area increases?

10. Find the discriminant of the quadratic equation $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

11. Write the nth term of the A.P. $\frac{1}{m}, \frac{1+m}{m}, \frac{1+2m}{m}$ ....

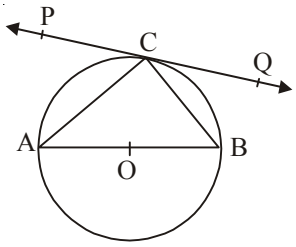
12. If (x + a) is a factor of $2x^2 + 2ax + 5x + 10$, find a.

13. What is the point of intersection of the line represented by 3x – 2y = 6 and the y-axis.

14. Find the coordinates of the point on y-axis which is nearest to the point (– 2, 5).

15. If the ratio of the height of a tower and the length of its shadow is $\sqrt{3} : 1$. What is the angle of elevation of the sun?

16. In figure PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and ∠CAB = 30°, find ∠PCA.



17. If a quadratic polynomial f(x) is factorisable into linear distinct factors, then the total number of real and distinct zeros of f(x) is .....

18. The distance between the points A ($\sin\theta - \cos\theta$, 0) and B(0, $\sin\theta + \cos\theta$) is .....

19. Sides of two similar triangles are in the ratio 4 : 9. The areas of these triangles are in the ratio .....

20. If $\tan A = \frac{5}{12}$, then the value of $(\cos A - \sin A)\ \text{cosec}\ A$ is ..... .

SECTION B

21. In a single throw of a pair of different dice, what is the probability of getting (i) a prime number on each dice (ii) a total of 9 or 11?

22. A hemispherical tank of diameter 3 m is full of water. It is being empied by a pipe at the rate of $3\frac{4}{7}$ litre per second. How much time will it take to make the tank half empty?

23. Cards marked with numbers 13, 14, 15, ... 60 are placed in a box and mixed thoroughly. One card is drawn at random from the box. Find the probability that number on the card drawn is:

(i) divisible by 5 (ii) a number which is a perfect square.

24. The length of the minute hand of a clock is 5 cm. Find the area swept by the minute hand during the time period 6 : 05 am and 6 : 40 am.

25. Solve for x and y:

$$\frac{4}{x} + 5y = 7\ ;\ \frac{3}{x} + 4y = 5$$

26. Show that any positive odd integer can be written in the form 6 m + 1, 6 m + 3 or 6 m + 5 where m is a positive integer.

SECTION C

27. Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

28. If $x = p\sec\theta + q\tan\theta$ and $y = p\tan\theta + q\sec\theta$, then prove that $x^2 - y^2 = p^2 - q^2$.

29. A is a point at a distance 13 cm from the centre O of a circle of radius 5 cm. AP and AQ are the tangents to the circle at P and Q. If a tangent BC is drawn at a point R lying on the minor arc PQ to intersect AP at B and AQ at C, find the

perimeter of the ΔABC.

30. Evaluate, without using trigonometric tables:

$\cot\theta \tan(90° - \theta) - \sec(90° - \theta) \operatorname{cosec}\theta + \sin^2 65° + \sin^2 25° + \sqrt{3} \tan 5° \tan 45° \tan 85°$

31. If α and β are zeroes of the polynomial $6y^2 - 7y + 2$, find the quadratic polynomial whose zeroes are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

32. Find a natural number whose square diminished by 10 is equal to five times of 8 more than the given number.

33. Prove that the area of the semi circle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the semi-circles drawn on the other two sides of the triangle.

34. An AP consists of 45 terms. The sum of the three middle most terms is 546 and the sum of the last three terms is 1050. Find the AP.

SECTION D

35. On selling a tea set at 5% loss and a lemon set at 15% gain, a crockery seller gains ₹ 7. If he sells the tea-set at 5% gain and the lemon-set at 10% gain, he gains ₹ 13. Find the actual price of the tea-set and the lemon-set.

36. Point P divides the line segment joining the points A(2, 1) and B(5, – 8) such that $\frac{AP}{AB} = \frac{1}{3}$. If P lies on the line $2x - y + k = 0$, find the value of k. Also find the distance between AP.

37. Draw an isosceles triangle ABC in which AB = AC = 6 cm and BC = 5 cm. Construct a triangle PQR similar to ΔABC in which PQ = 8 cm. Also justify the construction.

38. A person observes the elevation of a cloud from a point 60 metres above a lake as 30° and the angle of depression of its reflection in the lake as 60°. Find the height of the cloud.

39. Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in pond rise by 21 cm?

40. If the median of the following frequency distribution is 525, in the table given below, find the value of x and y. if total frequency is 100.

Variable	0–100	100–200	200–300	300–400	400–500	500–600	600–700	700–800	800–900	900–1000	Total
Frequency	2	5	x	12	17	20	y	9	7	4	100

ANSWERS KEY

1. (a) 4 **2.** (d) ogive **3.** (d) 4 **4.** (b) $\frac{1}{6}$

5. (a) 3 : 1 : 2 **6.** (a) 4 : 5

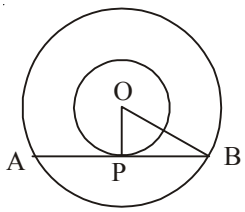
7. AD : AB = 1 : 3

$\Delta ADE \sim \Delta ABC$

$$\frac{DE}{4.5} = \frac{1}{3}$$

$\Rightarrow$ DE = 1.5 cm

8. In ΔOPB, $PB = \sqrt{(5)^2 - (4)^2} = 3$ cm



AB = 2 × PB = 2 × 3 = 6 cm

9. Let diameter of circle be d units.

Area of circle = $\frac{\pi d^2}{4}$ square units

Diameter of circle after increasing 40% = d + 40% of d = $\frac{14}{10}$ d unit

Increased area of circle = $\frac{196\pi d^2}{400}$ square units

% increase in area = $\frac{\frac{96\pi d^2}{400}}{\frac{\pi d^2}{4}} \times 100 = 96\%$

10. $D = (10)^2 - 4 \times 3\sqrt{3} \times \sqrt{3} = 64$

11. $a = \frac{1}{m}$ and $d = 1$

$a_n = \frac{1}{m} + (n-1) \times 1 = \frac{1 + m(n-1)}{m}$

12. $2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$

$a = 2$

13. $3(0) - 2y = 6$

$y = -3$

$\therefore$ required point is $(0, -3)$

14. $(0, 5)$

15. $\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

16. Join OC

$OA = OC$

$\Rightarrow \angle OCA = \angle OAC = 30°$

$\angle PCO = 90°$

$\angle PCA + \angle OCA = 90°$

$\therefore \angle PCA = 60°$

17. 2

18. $\sqrt{(\sin\theta - \cos\theta)^2 + (\sin\theta + \cos\theta)^2} = \sqrt{2}$ units

19. 16 : 81

20. $(\cos A - \sin A) \times \frac{1}{\sin A} = \cot A - 1 = \frac{12}{5} - 1 = -\frac{7}{5}$

21. (i) $\frac{9}{36} = \frac{1}{4}$ (ii) $\frac{6}{36} = \frac{1}{6}$

22. $\text{Time} = \frac{\frac{1}{2} \times \text{Volume of tank}}{\text{Water flown in 1 second}} = \frac{\frac{1}{2} \times \frac{2}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}}{\frac{25}{7000}} = 16\frac{1}{2}$ minutes

23. (i) $\frac{10}{48} = \frac{5}{24}$ (ii) $\frac{4}{48} = \frac{1}{12}$

24. Angle made by minute hand in 35 minutes $= \frac{35}{60} \times 360° = 210°$

$$\text{Area} = \frac{22}{7} \times \frac{(5)^2 \times 210°}{360°} = 45\frac{5}{6}\text{cm}^2$$

25. Let $\frac{1}{x} = a$

$\therefore \quad 4a + 5y = 7 \quad$ and $\quad 3a + 4y = 5$

Solve these equations, we get

$a = 3 \qquad$ and $\qquad y = -1$

$x = \frac{1}{3} \qquad$ and $\qquad y = -1$

26. Correct proof **27.** Correct proof

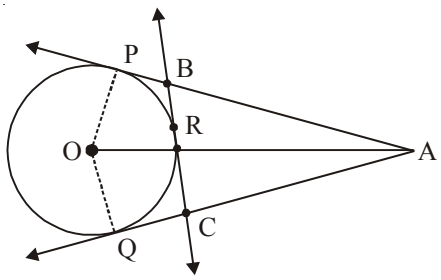
28. LHS $= x^2 - y^2 = (p \sec\theta + q \tan\theta)^2 - (p \tan\theta + q \sec\theta)^2$

$= p^2 \sec^2\theta + q^2 \tan^2\theta + 2pq \sec\theta \tan\theta - p^2 \tan^2\theta - q^2 \sec^2\theta - 2pq \tan\theta \sec\theta$

$= p^2(\sec^2\theta - \tan^2\theta) - q^2 (\sec^2\theta - \tan^2\theta)$

$= P^2 - q^2 = \text{RHS}$

29. OP $\perp$ AP $\Rightarrow \quad \angle OPA = 90°$



In Δ OPA, $OA^2 = OP^2 + PA^2$

$\Rightarrow \quad (13)^2 = (5)^2 + PA^2$

$\Rightarrow \quad PA = 12$ cm

Perimeter of $\Delta ABC = AB + BC + CA$

$= AB + BR + RC + CA = AB + BP + CQ + CA$

$= AP + AQ = 2AP = 2 \times 12 = 24$ cm

30. $\cot\theta \cdot \cot\theta - \operatorname{cosec}\theta + (\sin^2 65° + \cos^2 65°) + \sqrt{3} \tan 5° \cdot \tan 45° \cdot \cot 5°$

$= \cot^2\theta - \operatorname{cosec}^2\theta + 1 + \sqrt{3} \times 1 \times 1 = -1 + 1 + \sqrt{3} = \sqrt{3}$

31. $\alpha + \beta = \frac{7}{6}$ and $\alpha.\beta = \frac{7}{6}$

$S = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{7}{2}$

$P = \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{6}{2}$

$\therefore$ required polynomial is $k\left(x^2 - \frac{7}{2}x + \frac{6}{2}\right)$

Put $k = 2$, $x^2 - 7x + 6$

32. Let number be x

According to Question

$x^2 - 10 = 5(x + 8)$

$x^2 - 5x - 50 = 0$

$(x - 10)(x + 5) = 0$

either $x = 10$ or $x = -5$

but natural number is always positive. Hence, $x = 10$

33. Area of semicircle with diameter AB + Area of semicircle with diameter BC

$= \frac{\pi}{2}\left(\frac{AB}{2}\right)^2 + \frac{\pi}{2}\left(\frac{BC}{2}\right)^2 = \frac{\pi}{2}\left(\frac{AB^2 + BC^2}{4}\right) = \frac{\pi}{2}\left(\frac{AC^2}{4}\right)$

= Area of semicircle with diameter AC

34. $a_{22} + a_{23} + a_{24} = 546$

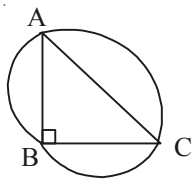
$a + 22d = 182$ ...(1)

$a_{43} + a_{44} + a_{45} = 1050$

$a + 43d = 350$ ...(2)

From (1) and (2), $a = 6$ and $d = 8$

$\therefore$ A.P. is 6, 14, 20,.....



35. Let actual price of the tea set be ₹ x

and actual price of the lemon set be ₹ y

According to the question,

$$-\frac{5}{100}x+\frac{15}{100}y = 7$$

$\Rightarrow$ $-x + 3y = 140$ ...(1)

$$\frac{5}{100}x+\frac{10}{100}y = 13$$

$\Rightarrow$ $x + 2y = 260$ ...(2)

From (1) and (2), x = ₹ 100 and y = ₹ 80

36. $P \leftrightarrow (3, -2)$

$2(3) - (-2) + k = 0$

$\Rightarrow$ $k = -8$

$$AP = \sqrt{(3-2)^2+(-2-1)^2} = \sqrt{10} \text{ units}$$

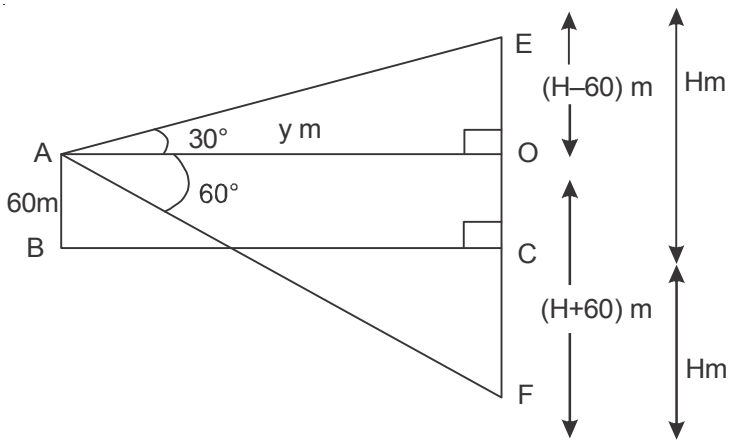
37. Correct construction and justification.

38. In ΔADE, $\frac{H-60}{y} = \tan 30°$ ...(1)

In ΔADF, $\frac{H+60}{y} = \tan 60°$ ...(2)

From (1) and (2), $H - 60 = \frac{2 \times 60 \tan 30°}{\tan 60° - \tan 30°}$

H = 120 m



39. Time required $= \dfrac{\text{Volume of water at rise of level 21cm}}{\text{water flown in 1 hour}}$

$$= \frac{50 \times 44 \times \frac{21}{100}}{\frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 15000} = 2 \text{ hours}$$

40.

Variable	Frequency	c.f.
0–100	2	2
100–200	5	7
200–300	x	7 + x
300–400	12	19 + x
400–500	17	36 + x
500–600	20	56 + x
600–700	y	56 + x + y
700–800	9	65 + x + y
800–900	7	72 + x + y
900–1000	4	76 + x + y
Total	100	

$76 + x + y = 100$

$x + y = 24$ ...(1)

$$525 = 500 + \frac{50 - (36 + x)}{20} \times 100$$

$x = 9$

from (1), $y = 15$

PRACTICE PAPER-II
CLASS: X
Mathematics (Standard)

Time : 3 hours **Max. Marks : 80**

General Instructions:

1. All the questions are compulsory.

2. The question paper consists of 40 questions and it is divided into four sections A, B, C and D.

3. Section A comprises of 20 questions carrying 1 mark each. Section B comprises of 6 questions carrying 2 marks each. Section C comprises of 8 questions carrying 3 marks each. Section D comprises of 6 questions carrying 4 marks each.

4. There is no overall choice.

5. Use of calculator is not permitted.

SECTION A

Question number 1 to 20 carry 1 mark each.

1. If n is a natural number then $9^{2n} - 4^{2n}$ is always divisible by:

(a) 5 (b) 13 (c) 5 and 13 (d) none of these

2. If the mean of the following distribution is 2.6 then the value of y is:

x_i	1	2	3	4	5
f_i	4	5	y	1	2

(a) 3 (b) 8 (c) 13 (d) 24

3. If the difference between the circumference and radius of a circle is 37 cm then using $\pi = \frac{22}{7}$, the circumference (in cm) of the circle is:

(a) 154 (b) 44 (c) 14 (d) 7

4. If $am \neq bl$ then the system of equations $ax + by = c$ and $lx + my = n$

(a) has a unique solution (b) has no solution

(c) has infinitely many solutions (d) may or may not have solution

5. The value of k for which the quadratic equation $x^2 - kx + 4 = 0$ have equal roots:

(a) 4, – 4 (b) 16 (c) – 4 (d) 4

6. The sum of three consecutive terms of an increasing A.P. is 51 and the product of 1st and 3rd of these terms is 273 then the third term is:

(a) 13 (b) 9 (c) 21 (d) 17

7. If $(k + 1) = \sec^2\theta (1 + \sin\theta)(1 - \sin\theta)$, find k.

8. If $(\text{cosec}\,\theta + \cot\theta) = x$ find $\text{cosec}\,\theta - \cot\theta$.

9. If a pole 6 m high casts a shadow of $2\sqrt{3}$ long on the ground then what is the sun's elevation?

10. State true or false and justify

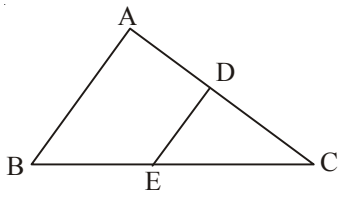
"If a die is thrown, there are two possible outcomes an odd number or an even number. Therefore the probability of getting an odd number is $\frac{1}{2}$".

11. State true or false and justify

"A driver attempts to start a car. The car starts or doesnot start is an equally likely outcome."

12. In an equilateral triangle, the lengths of the median is $\sqrt{3}$ cm, then find the length of the side of this equilateral triangle.

13. In the given figure of ΔABC, D and E are points on CA and CB respectively such that DE || AB, AD = 2x, DC = x + 3, BE = 2x – 1, CE = x find n.



14. Find the altitude of an equilateral triangle of side 8 cm.

15. Fill in the blanks:

If P(2, 4), Q(0, 3), R (3, 6) and S(a, b) are vertices of a parallelogram then the value of a + b is .....

16. Find K if the point P(2, 4) is equidistant from A(5, K) and B(K, 7).

17. Two tangents making an angle of 60° between them, are drawn to a circle of radius $\sqrt{2}$ cm, then find the length of each tangent.

18. If the sum and product of the zeros of the polynomial $ax^2 - 5x + c$ is 10 find a and c.

19. If α, β are zeros of $2x^2 - 5x + 1$ find a quadratic polynomial whose zeroes are 2α and 2β.

20. If radii of two concentric circles are 4 cm and 5 cm, then find the length of the chord of one circle, which is tangent to the other circle.

SECTION B

21. Prove $3 - \sqrt{5}$ is an irrational number.

22. Solve for x and y: $\frac{4}{x} + 5y = 7$, $\frac{3}{x} + 4y = 5$

23. A solid piece of iron is in the form of a cuboid of dimensions 4.4 m × 2.6 m × 10 m is melted to form a hollow cylinder of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.

24. In the following data, find the values of p and q. Also find the median class and modal class.

C.I.	Frequency	Cumulative frequency
100 – 200	11	11
200 – 300	12	p
300 – 400	10	33
400 – 500	q	46
500 – 600	20	66
600 – 700	14	80

25. If $7 \sin^2\theta + 3 \cos^2\theta = 4$, then find value of $\tan\theta$.

26. A box contains cards numbered from 13, 14, 15, ..., 60. A card is drawn at random from the box. Find the probability that the number on the drawn card is

(i) divisible by 2 or 3 (ii) a prime number

SECTION C

27. Show that the cube of any positive integer is of the form 9 m, 9 m + 1 or 9 m + 8.

28. Find all zeroes of the polynomial $2x^4 - 10x^3 + 5x^2 + 15x - 12$ when its two zeroes are $\sqrt{\frac{3}{2}}$ and $-\sqrt{\frac{3}{2}}$.

29. Solve for x : $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$, $x \neq 1, -2, 2$.

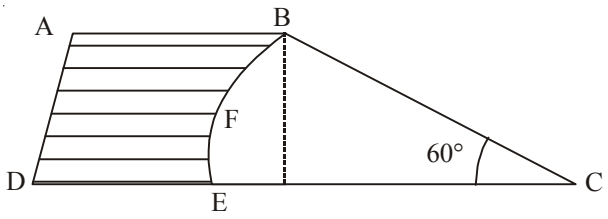
30. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

31. If an isosceles triangle ΔABC in which AB = AC = 6 cm is inscribed in a circle of radius 9 cm, find the area of the triangle.

32. In an A.P. of 50 terms, the sum of first ten terms is 210 and the sum of last 15 terms is 2565. Find the A.P.

33. Find the value of : $\left(\frac{3\tan 41^\circ}{\cot 49^\circ}\right)^2 - \left(\frac{\sin 35^\circ \sec 55^\circ}{\tan 10^\circ \tan 20^\circ \tan 60^\circ \tan 70^\circ \tan 80^\circ}\right)^2$

34. In the given figure ABCD is a trapezium with AB || DC and $\angle BCD = 60^\circ$. If BFEC is a sector of a circle with centre C and AB = BC = 7 cm and DE = 4 cm then find the area of the shaded region: $\left(\pi = \frac{22}{7}, \sqrt{3} = 1.732\right)$



SECTION D

35. The angle of elevation of a cloud from a point 60 m above a lake is 30° and the angle of depression of the reflection of cloud in the lake is 60°. Find the height of the cloud.

36. The height of a cone is 30 cm. A small cone is cut off at the top of a plane parallel to

the base. If its volume is $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made?

37. Draw a Δ ABC, with side BC = 7 cm, $\angle B = 45°$, $\angle A = 105°$. Then construct a triangle whose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC.

38. The distribution given below show the marks of 100 students of a class:

Marks	No. of students
0–5	4
5–10	6
10–15	10
15–20	10
20–25	25
25–30	22
30–35	18
35–40	5

39. Find the value (s) of k for which the points (3k – 1, k – 2), (k, k–7) and (k – 1, – k – 2) are collinear.

40. A motor boat whose speed is 18 km/hr in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.

ANSWERS KEYS

1. (C) as $a^{2n} - b^{2n}$ is divisible by both (a + b) and (a – b).

2. (B) **3.** (B) **4.** (A)

5. (A) **6.** (C) **7.** 0 **8.** $\frac{1}{x}$

9. 60°

10. True A = {1, 3, 5}
B = {2, 4, 6}

$$P(A) = \frac{1}{2} = P(B)$$

11. False; Car will start or not is not always equally likely.

12. $a = 2, a \neq -2$ **13.** $\frac{3}{5}$ **14.** $4\sqrt{3}$cm **15.** $a + b = 12$

16. $K = 3$ **17.** 3 cm **18.** $a = \frac{1}{2}, c = 5$

19. $x^2 + 5x + 1 = 0$ **20.** 6 cm **21.** Prove by method of contradiction

22. $x = \frac{1}{3}, y = -1$ **23.** 112 m

24. $P = 11 + 12 = 23$

$q = 13$

Median class 400–500

Modal class 500–600

25. $\tan\theta = \frac{1}{\sqrt{3}}$ **26.** (i) $\frac{32}{48} = \frac{2}{3}$ (ii) $\frac{12}{48} = \frac{1}{4}$

27. Use Euclid's division lemma

28. 4, 1 **29.** $x = -5, x = \frac{6}{5}$ **30.** Proof of theorem

31. $8\sqrt{2}$ cm^2

32. $S_{10} = 210$

$S_{50} - S_{35} = 2565$

$d = 4, a = 3$, A.P. 3, 7, 11, ...

33. $\frac{26}{3}$ **34.** 28.89 cm^2 **35.** h = 120 m

36. h = 20 cm **37.** Construction

38. Ogive median 24 (approximates) from graph

39. $K = 0, 3$ **40.** 6 km/hr

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